MINIMUM COST DESIGN OF TWO-HINGED HIGHWAY ARCH RIBS

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MINIMUM COST DESIGN OF TWO-HINGED

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SUMMARY

The objective of this paper is to investigate the minimum cost design of a two-hinged highway arch rib having a box section. The investigation considers variation of both arch geometry and rib section size, and the objective function includes the cost of the webs, flanges, diaphragms and longitudinal stiffeners. The governing stress from the loads is assumed to be equal to the allowable stress, and the objective function is assumed to be unimodal.

The results of the investigation include the following:

1. A discussion of the design procedure is given.

2. An objective function surface for the cost of a 310 ft parabolic arch rib is given along with several other surfaces for parameters influenced by the designs for the objective surface.

3. Several factors influencing the least cost design are discussed.

4. A discussion of a procedure for reducing excessive live load deflections is made.

5. A discussion of the search methods studied is included and a recommended design procedure is presented.

The following conclusions can be made with regards to the investigation:

1. The cost objective function for the 310 ft parabolic arch is strongly unimodal within the region of practical designs.

2. Convergence of a particular design for fixed values of the independent

variables, defined as noise, and possible multiple values of the objective function are the most critical problems relating to the design of the two-hinged, non-prismatic arch.

3. Excessive live load deflections can be reduced by adjusting web depth and arch height keeping a design where the governing stress from the loads is equal to the allowable; however, if the correction for excessive live load deflection is large then increasing the flange thickness in addition to adjusting the web depth and arch height may be more economical.

CHAPTER I

INTRODUCTION

Traffic loads on bridge structures may be supported by several structural configurations. Some of the structural configurations used for bridges are girders, trusses, arches and suspension cables. The arch in bridges may be used as a large barrel arch culvert, an open spandrel arch or a truss arch. The materials of construction, geometric shape of the arch and support conditions will vary. For the purposes of this investigation the two-hinged, open spandrel arch rib having a rectangular box section has been selected.

In addition to being esthetically pleasing the arch rib is competitive with other structural configurations for some span ranges and site conditions; however, the total cost of the arch rib is influenced considerably by certain design variables as is shown later. The objective of this thesis is to establish an approved design procedure for the two-hinged highway arch rib using a box section and to show the effects of the important parameters that influence the final design.

The objective is accomplished in the following manner:

(1) A computer program for the two-hinged highway arch rib is written to determine the section size requirements at preselected control points (defined as design points in Chapter II) according to the AASHO (American Association of State Highway Officials) Specifications [64].* The computer program produces

*Numbers in [ ] refer to bibliographical reference numbers, and numbers in ( ) refer to equations.

section sizes through a repeated analysis redesign procedure so that the rib section sizes converge to stable values that satisfy all of the stress requirements. The maximum permitted stress and the maximum stress resulting from the applied loads are discussed in more detail in Chapter II, and the results of the convergence to stable section sizes defined as "noise" is discussed in Chapter III.

(2) An objective function is established by using the section size requirements to compute a weight related total cost of the arch rib. If the price of materials is the same for all of the rib materials then the minimum cost is also the minimum weight.

(3) The total cost of the arch rib is used to search for the minimum cost and to study the influence of certain important parameters on this minimum cost.

(4) The investigation establishes a design procedure which leads to a minimum cost design.

Up until the last 50 to 100 years craftsmen and carpenters did much of the structural design. In some of these cases, these craftsmen constructed models and either stood upon them or loaded them in some other way to demonstrate that their idea was sound.

Within the last 100 years, however, the analysis and design of structures continued to become more theoretical and mathematically complex. Among those who contributed most to the theoretical development of structural analysis are Maxwell who wrote his paper concerning the stiffness of framed structures in 1864 [47], Green who lectured on the moment-area method at the University of Michigan in 1872 [51], Maney who developed the slope deflection method in

1915 [73] and Hardy Cross who developed the moment distribution method circa 1932 [15]. Some of the early investigators using matrix methods for the solution of large structures with the electronic digital computer are Wehle and Lansing in 1952 [70], Langfors in 1952 [43] and in 1953 [44] and Denke in 1954 [19]. Some of the early investigators who introduced the concept of the finite element method in continuum mechanics are Turner, Clough, Martin and Topp in 1956 [67].

Using these methods for the analysis of statically indeterminate structures, analysis could be made on a more theoretical basis. However, for many large structures, the analysis was an overwhelmingly difficult task by hand. This led to the further development of approximate methods of solution where trusses were assumed to be pin-connected at the joints and frames were assumed to be hinged at assumed member inflection points. These and other simplifications made the structure statically determinate.

With the continuing development and adaption of engineering problems to the computer, the engineer has been relieved of much of the time consuming computational work. He can now compare the results of several designs, even for large structural systems, to arrive at what is apparently the best of several designs.

In general, engineers have always attempted to determine the most economical design for a particular situation. Michell placed the search for the most economical design on a highly theoretical basis in "The Limits of Economy of Materials in Framed Structures" in 1904 [48]. New interest has been generated in "Michell" structures within the last few years because a Michell structure does furnish the absolute minimum weight structure for a given fixed load system. Some of the researchers of Michell structures include Hemp in 1958 [32], A. S. L. Chan in 1960 [10], H. S. Y. Chan in 1963 [11] and Ghista in 1966 [27].

Within the last fifteen years, the computer has been used more and more in order to find the most economical design. Much of this research is still on such a theoretical basis that it is not directly usable by the practicing engineer; however, the past research does indicate the direction which future research should take.

Many different structural systems (beams, columns, trusses, frames, slabs, plates, shells and prestressed concrete structures) have been considered for optimum design; however, a large portion of this research has been directed toward truss and unbraced frame type structures. A few of the researchers who have considered trusses and/or frames are Prager in 1956 [53], Heyman and Prager in 1958 [33], Brotchie in 1964 [7], Hill in 1966 [34], Rubinstein and Karagozian in 1966 [60], Bigelow and E. Gaylord in 1967 [3], Romstad and Wang in 1968 [59], Dobbs and Felton in 1969 [20], Johnson and Brotton in 1969 [37], Kavlie and Moe in 1971 [39], LaPay and Goble in 1971 [45], and Vanderplaats and Moses in 1972 [68].

Five papers which solve problems very useful to the Civil Engineer are (1) "Optimum Design of Constant Depth Plate Girders" by Razani and Goble in 1966 [57], (2) "Optimum Design of Mixed Steel Composite Girders" by Goble and DeSantis in 1966 [28], (3) "A Method for the Optimum Design of Simple Span Deck and Girder Highway Bridges" by Cornell, Ho and Ehrlich in 1967 [14], (4) "Development of Automated Optimum Structural Design Systems" by Corey in 1968 [13], and (5) "Optimal Design of Prestressed Concrete Poles" by Thakkar and Bulsari in 1972 [65]. The optimum design method used in papers [13, 28 and 57] is one of the search methods, and the method used in papers [14 and 65] is nonlinear programming.

Most of the research has been directed toward finding the most economical member sizes for a given structural geometric configuration. Very little research has been done toward finding the most economical geometry. Michell type structures are directed toward this end in addition to producing the most economical member sizes. Some of the researchers who have considered the optimum geometry of trusses are Dobbs and Felton in 1969 [20] and Vanderplaats and Moses in 1972 [68]. Optimum framing layouts have been considered by Lewis whose research was published in 1966 in Computers in Engineering Design Education, Vol. III [12], Lipson and Russell in 1971 [46], and Grundy in 1971 [30]. Optimum geometry of shell structures was considered by Smith and Wilson in 1971 [63].

Although the classical calculus methods of minimization are quite old, many numerical procedures of minimization are relatively new. Some investigators into numerical minimization are listed below. In 1944 Curry [16] discussed the method of steepest descent which was originally proposed by Cauchy in 1847. Dantzig and his colleagues at the RAND Corporation developed the simplex method for solving linear programming problems in 1947; however, a good explanation of the simplex method is given by Dantzig in 1956 [17]. Kiefer developed the Fibonacci search and suggested the golden section search in 1953 [41]. Bellman developed dynamic programming in the 1950's [2]. Kelly developed the cutting plane method of solving nonlinear programming problems in 1960 [40]. Hooke and Jeeves developed the direct search and pattern search in 1961 [36]. Powell developed the method of conjugate directions in 1964 [52]. Although the method of conjugate gradients had been proposed previously to solve a set of linear equations, Fletcher and Reeves proposed the method of conjugate gradients to solve optimization problems in 1964 [23]. Davidon originally proposed the variable metric method for optimization problems for which the derivatives could be computed in 1959 [18], and Fletcher and Powell sharpened the method in 1963 [22]. Box discussed the problem of constrained optimization in 1965 [5]. Fox discussed the use of penalty functions for constrained optimization in a book written in 1971 [24]. Kavlie and Moe used penalty functions in 1971 [39], and Jones and Hague used penalty functions in 1972 [38]. Bellman in 1957 [2]; Mitten and Nemhauser in 1963 [49]; Wilde in 1965 [72]; and Kirsch, Reiss and Shamir in 1972 [42] discussed ways of breaking large problems down into several smaller ones. Gomory in 1960 [29]; Toakley in 1968 [66]; Reinschmidt in January, 1971 [58]; Cella and Logcher in January, 1971 [8]; and Cella in March, 1972 [9], discussed and proposed methods for optimizing discrete variable problems.

The field of optimization continues to be a good field in which to do research. Larger and more complex structural solutions are being produced. For bridge structures the plate girder was designed by Razani and Goble in 1966 [57], and the composite girder was designed by Goble and DeSantis in 1966 [28]. Both of these design procedures were produced for the computer using influence lines in the analysis and search techniques in the optimization. As far as the author knows no one has investigated the arch for highway bridges, and for this reason the two-hinged highway arch rib has been selected for this investigation.

CHAPTER II

DESIGN PROCEDURE

Formulation of the Problem

A cost objective function based upon the weight of the arch rib is computed. Although the total cost of the arch rib is dependent upon many variables, the rib cost (COST) may be expressed mathematically as a function of the following variables (see Figs. 1 to 4).

$$\text{COST} = f(p_1, p_2, d, w, t_w, t_f, L, S, H, Y) \tag{1}$$

where p_1, p_2 = fabricated, in place price of materials (p_1 = price for webs and flanges, p_2 = price for stiffening elements). d, w, t_w, t_f = section size dimensions (Fig. 4, d = web depth, w = flange width, t_w = web thickness and t_f = flange thickness). L = arc length which is dependent upon the geometric shape (Fig. 1, parabolic; Fig. 2, circular; Fig. 3, straight segmented) and the geometric dimensions (S = span length, H = arch height for parabolic and circular arches, Y = vertical elevations for the straight segmented arch).

As shown in Fig. 1 the arch is divided into several (NP, including zero) design control points called "design points." The design points must include both supports plus all column locations called "column points" (NC, including zero) which are on the arch rib; however, any number of additional design points which may include splice points may be located and specified by the designer. These design points are located by measuring horizontally from the left, high support; and the design point locations remain fixed throughout the design procedure. As a result of the fact that there are several design points, the flange thickness values (t_f) represent many more variables than one; and the vertical elevations (Y) for the straight segmented arch also represent many more variables than one. The actual number of flange thickness (t_f) design variables is one less than the number of design points since the flange thickness used when computing the cost is the largest value at either end of the arc segment connecting the two design points. The actual number of vertical elevation (Y) design variables is two less than the number of design points since the supports are not permitted to move.

Considering equation (1), the cost is a function of many parameters, but some of these parameters are either fixed or dependent upon the remaining independent variables. The prices of materials (p_1, p_2) are selected by the designer and remain constant throughout the design process. The flange width (w) is dependent upon the web depth (d) through a width-to-depth ratio selected by the designer, and the ratio remains constant throughout the design procedure. This ratio can be changed, and the program resubmitted if the designer so desires. The web thickness (t_w) is expressed in terms of the web depth (d) according to the AASHO Specifications [64] by a web depth-to-thickness ratio. If the geometry of the arch rib has been selected as either parabolic or circular then the vertical elevation (Y) to each design point is dependent upon the arch height (H); however, if the geometry of the arch rib has been selected as straight segmented then the arch height (H) is not required since the vertical elevations (Y) to each design point describe the geometry. For the parabolic and circular arch, the following independent variables appear in the objective function

$$COST = f(d, H, t_{fo}, t_{f1}, t_{f2}, \ldots t_{fNP-1}) \ , \qquad (2)$$

NP = design points; and for the straight segmented arch, the following independent variables appear in the objective function

$$COST = f(d, t_{fo}, t_{f1}, t_{f2}, \ldots t_{fNP-1}, Y_1, Y_2, Y_3, \ldots Y_{NP-1}). \qquad (3)$$

For an optimization procedure, equations (2) and (3) represent the design objective. In a true optimization procedure the design objective and the constraints would be handled as by Jones and Hague in June, 1972 [38]. Such an optimization procedure can consider both the geometric and behavioral constraints as a part of the objective function through the use of a penalty function. Geometric constraints are associated with space restrictions while behavioral constraints are associated with the response of the structure to loads (limitations on displacements and stress levels). The penalty function works in such a way that when one of these constraints is violated the objective function is penalized in order to reduce the possibility of a second or continued violations.

The procedure used in this thesis to seek a minimum cost is based upon a comparison of designs which meet all of the constraint requirements. Both geometric and behavioral constraints exist for the two-hinged arch. The stress constraint which is a behavioral constraint is discussed in the following paragraph, and the remaining constraints are discussed later in this chapter.

Usually specifications place an upper bound on the normal stress so that

$$\sigma_e \leq \sigma_a \tag{4}$$

where σ_e = stress from applied loads and σ_a = allowable stress. For this thesis the assumption is made that

$$\sigma_e = \sigma_a . \tag{5}$$

This assumption simplifies the objective function (equation 2 or 3) considerably since the flange thickness variables (t_f, member section sizes) are eliminated from being independent variables. The remaining independent variables in the objective function for the parabolic and circular arch are

$$\text{COST} = f(d, H) \tag{6}$$

where d = web depth and H = arch height, and the independent variables in the objective function for the straight segmented arch are

$$\text{COST} = f(d, Y_1, Y_2, Y_3, \ldots Y_{NP-1}) \tag{7}$$

where Y = vertical elevation of design points and NP = number of design points.

An explicit equation for the cost estimate for both equation (6) and (7) is given as

$$\text{COST} = 2p_1\rho t_w dL + 20p_1\rho w \sum_{i=o}^{NP-1} t_{fi}\Delta L_i + p_2\rho C_1 d t_w L \tag{8}$$

where p_1 = price of steel per pound for webs and flanges, p_2 = price of steel per pound for diaphragms and longitudinal stiffeners, ρ = density of steel, C_1 = constant, t_w = thickness of web, d = depth of web, w = width of flange, t_f = thickness of flange, ΔL = 1/10th arc length between design points and L = total length of arc. Both ΔL and L are dependent upon the geometrical properties of the arch rib which are (1) type of arch (parabolic, circular or straight segmented), (2) span length (S), (3) arch height (H), (4) or elevation (Y) of design points.

In order to seek the minimum of the objective function (either equation 6 or 7) three steps are used, and these three steps are as follows: reduction of noise in the objective function, comparison of designs to search for improvement and final convergence to the minimum cost design. These steps are discussed in the following three paragraphs.

1. Noise in the objective function is reduced sufficiently to make a comparison of designs and to establish a search pattern. Reduction of noise is accomplished through a series of analysis redesign cycles for fixed values of the independent variables in equation (6) or (7) until the flange thicknesses (t_f) converge to stable values where the convergence is measured by the change in total rib cost. This type of a convergence problem results from the fact that the two-hinged arch is statically indeterminate, and this convergence problem is defined and treated as "noise" in this thesis because of a similar type of problem discussed by Wilde [71].

2. Once the noise is reduced sufficiently for one set of independent variables (equation 6 or 7) then a new set of variables is selected by making a

comparison of designs and using one of the search techniques discussed later. The search is continued with new sets of independent variables until the least cost design is approached.

3. Convergence to the minimum cost design along the objective function surface is determined by tolerance values placed upon the independent variables (equation 6 or 7) by the designer. A tolerance for total cost is selected by the designer to control noise, and as the minimum cost design is approached the convergence requirement for noise becomes more stringent so that the noise will not control the search direction.

Application of Search Methods

Several search methods were used at various stages during the investigation. These search procedures are discussed in the following few paragraphs.

The exhaustive search [12] is used in all cases to find the maximum and minimum live load stress, deflection and shear (moment and thrust are computed for the maximum and minimum stress condition). In addition to this the exhaustive search is used to find the governing stress and governing shear condition by searching all load combinations. In spite of the faults of the exhaustive search, it is still the best method to find the governing load conditions. This is true for three reasons: the influence lines (discussed later in this chapter) are not unimodal as required by most search methods; the influence lines have a finite number of loading points (columns); and the absolute maximum and minimum condition is desired.

The first search procedure to be used to seek the minimum of the objective function (equation 6 or 7) was the univariate search [12] using the Fibonacci procedure developed by Kiefer in 1953 [41]. For the parabolic and circular arch, equation (6), only one cycle involving arch height (H) and web depth (d) was performed in order to save computer time. First the arch height was adjusted, and then the web depth was adjusted. If stress controlled the design, these two steps were assumed to be sufficient; however, if live load deflection was found to be excessive, then a third step was made in order to reduce live load deflection. For the straight segmented arch, equation (7), the elevation (Y) of each design point was adjusted in sequence, and the cycle of adjustments was continued until the arch geometry stabilized. Next the web depth was adjusted. If live load deflection was excessive then it was reduced. A flow diagram for the single cycle univariate search procedure is shown in Fig. 5.

A single cycle of the univariate search neglects the interdependence between independent variables, and such a limited application of the univariate search will not find the least cost design if the interdependence between independent variables is strong. In order to improve the search for the minimum cost design three multivariate search procedures which consider the interdependence between design variables were examined for the parabolic and circular arch, equation (6). These three search procedures, in the order that they were studied, are the gradient [71], conjugate direction [24, 52] and univariate [12] search using several complete cycles where both independent variables are considered during each cycle. One of the three searches is used to seek the low point of the objective function assuming that stress rather than deflection controls the design; however, if live load deflection is found to be excessive then the least cost design is searched for along the boundary where live load deflection first becomes excessive. This boundary search is similar to a search along a cusp or a sharp ridge formed by two intersecting surfaces. The ridge search is discussed by Wilde [71]. Once the search direction is established by one of the multivariate search techniques a stepping procedure described in the following references [12, 24, 71] is used to find the low point of the curve in that direction. A flow diagram for the multivariate searches if given in Fig. 6.

General Design Computations

The design computations discussed correspond to the computer program of Appendix F which considers only the parabolic and circular arch. A reasonably good understanding of the order of the design computations can be obtained by referring to the flow diagram of the computer program which is discussed in Appendix E. The analysis redesign procedure is used where an initial design (consisting of web depth, flange width and flange thickness plus arch height) is assumed by the designer and submitted to the computer as input. The designer selects span length, arch type (parabolic or circular), rib cross sectional properties, horizontal location of the columns and design points, loads, strength properties of the steel, prices per pound of steel and tolerance values. The computer then alters the design using a search procedure until the least cost design is approached.

Assuming that all of the data have been read into the computer and all of the computer initialization of variables has taken place, the computations of the analysis redesign procedure are briefly described in the following steps.

1. The arch geometry consisting of distances, slopes and arc segment lengths plus rib section properties consisting of area and moment of inertia are computed.

2. The magnitudes of the influence line ordinates are computed.

3. The internal dead load moment and thrust are computed at all design points.

4. The maximum and minimum internal stress due to live load plus impact and the corresponding moment and thrust values are computed at all design points.

5. The internal moments and thrusts due to longitudinal force, temperature change, wind and earthquake are computed at all design points.

6. The governing stress, moment and thrust combinations are determined using equations (42) to (48) which are taken from the AASHO Specifications [64] and are described later.

7. Simultaneously while computing the governing stress, the allowable stress is computed for each design point using equations (49) to (53) which are taken from the AASHO Specifications [64] and are described later.

8. The flange thickness is adjusted so that the resulting governing stress is equal to the allowable stress, equation (5).

9. The total weight of the arch rib is computed. This weight includes

two quantities which are the total weight of the flanges and webs plus the approximate weight of the stiffeners and internal diaphragms.

10. The total cost of the arch is computed using the appropriate price per pound of steel for each part.

11. The total cost of the rib is compared with the cost of previous designs in order to reduce noise and to seek the least cost design. Each time the design is repeated in order to reduce noise and each time the design is made again because of a change in arch height and/or web depth the procedure begins at step one and continues through step eleven.

12. As soon as the search has converged and the tolerance values are satisfied assuming that stress controls the design then the live load deflections are computed at all column points. If any of the live load deflections are excessive, then the web depth and arch height are adjusted until the low point of the objective function is found on the boundary of excessive deflections.

13. For the last few designs the secondary stresses caused by deflections are considered. For the final design, shear force calculations are made for all loads, and the governing shear force is found at all design points.

Arch Shape and Rib Equations

The computation of the geometry and cross sectional properties of the arch rib are made through the use of the equations listed on the following pages. This is accomplished in the following order.

1. The computer takes the left support as the high support as shown in

Figs. 1 and 2. The X-Y coordinate axes are placed here, and all distances and rib slope angles are measured according to this coordinate system.

2. The horizontal position of the crown (Figs. 1 and 2) is located by computing the long side (S_L) and the short side (S_s) for the arch span (S). If the arch is symmetrical then $S_L = S_s$, and the crown of the arch is located at the center of the span. If the arch is not symmetrical, then S_L for the parabolic arch is found by

$$S_L = \frac{2S}{\sqrt{\frac{H}{H+\Delta H}} + 1} \tag{9}$$

where H = height, ΔH = difference in support elevations; and S_L for the circular arch is found by

$$S_L = \frac{2(H+\Delta H)}{\Delta H}\left[S + \sqrt{\frac{S^2 H+H(\Delta H)^2}{H+\Delta H}}\right]. \tag{10}$$

The distance S_s is found by

$$S_s = 2S - S_L . \tag{11}$$

3. The vertical distance (y) measured from the left, high support to the arch is found by using the given horizontal distance (x). This vertical distance for the parabolic arc is computed by the equation

$$y = H - k\left[\frac{S_s}{2} - x\right]^2 \tag{12}$$

where

$$k = 4H/(S_S)^2 = 4(H + \Delta H)/(S_L)^2. \tag{13}$$

The vertical distance for the circular arc is computed by the equation

$$y = H - (S_S/2 - x) \tan(\theta/2) \tag{14}$$

where the arc slope angle theta (θ) is found by

$$\tan\theta = (S_S/2 - x) \Big/ \sqrt{R^2 - (S_S/2 - x)^2}\ , \tag{15}$$

and the radius of the arc (R) is found by

$$R = S_S^2/(8H) + H/2. \tag{16}$$

4. The rib slope angle theta (θ) for the parabolic arc is found by

$$\tan\theta = (8H/S_S^2)(S_S/2 - x)\ , \tag{17}$$

and the slope angle theta (θ) for the circular arc is found by equation (15).

5. The arc length (L) measured from the arch crown for the parabolic arch is computed by

$$L = (S_S/2 - x)(SQ/2) + \frac{k^2(S_S/2 - x)^3}{4} \ \mathrm{Ln}\left[\frac{2 + SQ}{k(S_S/2 - x)}\right] \tag{18}$$

where

$$SQ = \sqrt{4 + k^2(S_s/2 - x)^2} \ . \tag{19}$$

The arc length measured from the arch crown for the circular arch is computed by

$$L = R\theta . \tag{20}$$

6. The arc length between design points is computed and quite arbitrarily divided into ten equal lengths (ΔL) to simplify later numerical integration by Simpson's rule.

7. Both horizontal and vertical distances and slope angles are computed at the ends of each of these increments of arc length. For the parabolic arch this involves an iterative approach by Newton's method for finding roots. An equation similar to equation (18) and its derivative are used to find the horizontal distances, and equations (12) and (17) are used to find the vertical distances and the slope angles. The distances and slope angles for the circular arch can be computed much easier and in a more direct manner by using equations (14), (15) and (20) or modifications of these equations.

8. The area (A) of the rib section (Fig. 4) is

$$A = 2dt_w + 2wt_f, \tag{21}$$

and the moment of inertia of the rib section is

$$I = t_w d^3/6 + wt_f(d+t_f)^2/2 + wt_f^3/6 \tag{22}$$

where d = web depth, t_w = web thickness, w = flange width and t_f = flange thickness. The designer submits the web depth, flange width and the initial flange thickness at each design point. The computer finds the web thickness by

$$t_w = d\sqrt{550F_y/1.25}\ /14400 \tag{23}$$

where F_y is the yield stress of the steel in ksi. This equation (23) for web thickness is taken from the AASHO Specifications [64]. The computation for the new flange thickness is discussed under the heading of Flange Thickness Adjustment in this chapter. For any single arch design the web depth, flange width and web thickness remain constant; however, the flange thickness is permitted to vary at all design points. The section size of the elements of arch between design points remains constant and is governed by the requirements at the design points at each end of the element.

Influence Lines

The analysis and redesign of the arch rib is accomplished through the use of influence lines. Influence lines are computed for the two reaction components, moment, thrust, shear, stress and deflection along the arch. For a hypothetical case having 20 design points and 10 column points a total of 92 influence lines must be computed. These include two for reactions; 20 each for moment, thrust, shear and stress; and 10 for deflection. The ordinates for all of these influence lines must be computed at each column point.

First the ordinates of the influence lines for reaction components are computed, and then the ordinates for the remaining influence lines are computed using those for reaction components as a starting point. A discussion of the reaction components is made in the following few paragraphs. Next moment, thrust, shear and stress are computed. Deflection influence lines are discussed under the heading Deflections.

In order to compute the ordinates for the influence lines for the reaction components (H_A and V_A) one of the horizontal reaction components (H_A or H_B) is released making the structure statically determinate. A unit load located at a horizontal distance X_L and pointing vertically downward on the arch rib is moved along the arch and stopped at each of the column points (Fig. 7). With the unit load stopped at a particular X_L distance the statically indeterminate arch is solved by computing the horizontal reaction component ($H_A = H_B$) according to Parcel [51], by

$$H_A = H_B = \frac{\int \frac{M(y + \Delta Hx/S)}{EI}\, dL}{\int \frac{(y + \Delta Hx/S)^2}{EI}\, dL + \int \frac{1}{AE}\, dL} \tag{24}$$

where M = moment due to the unit vertical load at a horizontal distance x and a vertical distance y, ΔH = vertical difference in elevation between the two supports, S = span length, dL = increment of arc length, E = modulus of elasticity, I = moment of inertia and A = area. The equation (24) is solved by numerical integration according to Simpson's rule between design points. This distance between design points has been quite arbitrarily divided into ten equal increments of ΔL

in length as stated previously.

The vertical reaction component at the left support (V_A) is

$$V_A = [S - X_L - H_A(\Delta H)]/S, \tag{25}$$

and the vertical reaction component at the right support (V_B) is

$$V_B = 1 - V_A \,. \tag{26}$$

Equation (24) takes the deflection due to thrust into account when solving for the horizontal reaction component. Normally the deflection due to thrust may be deleted from equation (24) with no great sacrifice in accuracy; however, if the height-to-span ratio (H/S) is less than approximately 1/10 the deflection due to thrust should be considered. The computer program uses this portion of the equation if the H/S ratio is less than or equal to 0.15.

A study was made to determine the effect of thrust deflections upon the resulting horizontal reaction component (H_A) for a unit load at the crown. The results of the computations are shown in Table 1. These computations refer to the arch of Fig. 8 and equation (27) which is for a symmetrical parabolic arch whose moment of inertia (I) and area (A) vary as the secant of the slope angle of the arch rib. The equation for the horizontal reaction component (H_A) has been taken from reference [25] and is

$$H_A = \frac{5S}{8H}\left[\frac{k_c - 2k_c^{\,3} + k_c^{\,4}}{1 + \dfrac{15\,I}{8A\,H^2}}\right] \tag{27}$$

where $$k_c = X_L/S. \tag{28}$$

The ordinates at column points for the moment (M), thrust (N), shear (V), stress (σ) and deflection (δ) influence lines for all design points are computed as follows. Fig. 9 is used to help derive equations (29) to (32). The ordinates for the moment (M) influence lines are computed by

$$M = -H_A(y - \Delta_{LL} - \Delta_{DL}) + V_A x - T(x - X_L) \tag{29}$$

where the maximum downward live load deflection is Δ_{LL}, the downward dead load deflection is Δ_{DL}, and the step function (T) is defined as

$$\begin{aligned} T &= 0 \text{ for } x \leq X_L, \\ T &= 1 \text{ for } x > X_L. \end{aligned} \tag{30}$$

The ordinates for the thrust (N) influence lines are computed by

$$N = H_A \cos\theta + V_A \sin\theta - T \sin\theta \tag{31}$$

where the slope angle of the arch rib is θ. The ordinates of the shear (V) influence lines are computed by

$$V = H_A \sin\theta - V_A \cos\theta + T\cos\theta. \tag{32}$$

The ordinates for the stress (σ) influence lines are computed by equation (41), and the ordinates for the deflection (δ) influence lines are computed by equation (57).

As may be noted from the drawings (Figs. 25 to 29) found in Appendix B, the influence lines for an arch bridge are slightly different from those of the theoretical arch. Some of these differences are:

1. The influence lines for an arch bridge are usually longer than the span of the arch rib since the influence lines are carried to the next support beyond the hinged end of the arch rib span.

2. The magnitudes of the influence lines are computed only at the column points since all deck loads are transferred through the columns.

3. The magnitude of all influence lines is zero at the end columns since these columns are located off of the arch rib and since a load at this point does not affect the stress in the arch rib.

4. The influence lines are made up of straight lines since the bridge deck is assumed to be simply supported between the columns and since the columns are assumed hinged at both ends.

Dead Load

The dead loads which are carried by the arch rib consist of (1) the rib weight, (2) the column weight and (3) the deck weight. The deck weight is usually by far the largest of these three loads.

Rib forces resulting from dead loads are computed as follows:

1. The rib dead load forces are assumed to result from a uniformly distributed load which is equal to the average weight of the rib; therefore, the rib forces are computed by multiplying the average rib weight by the area under the influence lines which is determined by numerical integration according to the

trapezoid rule.

2. The rib forces caused by the weight of the columns are found by multiplying the total column weight by the magnitude of the influence line at the columns. The total weight of the column is found by multiplying the column weight per foot which is assumed by the designer by the column length which is found by the computer.

3. The rib forces caused by the deck weight are found by multiplying the deck dead load forces delivered to the columns by the ordinates of the influence lines at these points. These forces applied to the columns by the dead weight of the deck are computed by the designer and submitted to the computer as input. This procedure makes the program slightly more flexible since the designer can easily account for variable column spacing, and he can account for added loads at some or all columns due to utilities or other loads.

Live Load

Both the maximum and the minimum live load stress must be computed at each design point for the three live loads (Fig. 10) which are interstate (sometimes called military loading), truck and lane loading. These live loads are discussed in more detail later. Computing the live load stresses and forces requires a large portion of computer time. The difficulty arises in determining the location of the live load which will produce the extreme stresses. This is true since the stress in the arch rib is caused by both moment and thrust; simply finding the point of the maximum moment will not be sufficient. A positive stress (compressive) might result even though the load is located within a negative region of the moment influence line. Thrust in the arch will always produce a compressive stress; however, moment can cause either tension or compression.

Locating the point which produces the extreme live load stress is greatly simplified by the use of stress influence lines. Typical influence lines for stress are shown in Appendix B (Fig. 28). These influence lines consist of straight lines between column points because of the manner in which the bridge deck load is transferred to the arch rib. Once the stress influence lines are drawn, the location of the live load which will produce the extreme stresses, consisting of both maximum and minimum stresses, can readily be determined.

There are two reasons for computing both the maximum and the minimum stress. (1) These two stresses must be computed at each design point in order to make the allowable fatigue stress computations. (2) These two stresses must be computed and combined with stresses from other loads in order to determine the governing stress combination.

Impact is included in all live load stress computations except those for sidewalk live load according to the AASHO Specifications [64]. The impact coefficient (I_m) is computed by

$$I_m = \frac{50}{S + 125} \leq 0.30 \qquad (33)$$

where S = arch span in feet.

The three live loads are discussed in the following paragraphs in the following order: (1) interstate loading, (2) truck loading and (3) lane loading.

The interstate loading of Fig. 10a was adopted as a loading for interstate bridges to account for some military loads. This load consists of two equal 24 kip axle loads spaced four feet apart. The loaded columns which produce the extreme stresses are found by an exhaustive search of all of the column point ordinates of the stress influence lines. One axle load is placed four feet away in the direction which produces the largest extreme stress. The designer computes the magnitude of the axle load (P_I) which is delivered to the arch. This load may be zero if interstate loading is not required for this particular design. If $P_I = 0$, this portion of the computer program is by-passed.

The truck loading which is shown in Fig. 10b may be one of five different types as follows: (1) H10, (2) H15, (3) H20, (4) HS15 or (5) HS20, AASHO Specifications [64]. All H loadings consist of a small and a large axle load which are spaced at 14 ft apart. The large axle load which is called P_{T1} is equal to four times the smaller axle load, and both loads add up to the number following H in tons. For example, the total load of the H20 truck is 20 tons or 40 kips. All HS loadings consist of one additional load which is called P_{T2} and is equal in magnitude to P_{T1}. It is placed at a variable distance of 14 to 30 ft from P_{T1}. The total load of the HS20 truck is 36 tons or 72 kips.

The truck loading is more difficult to work with than the interstate loading for two reasons: (1) the truck loading is unsymmetrical and (2) the distance between the two large axle loads of the HS loading is variable. The first problem is solved by assuming the truck to travel in both directions and solving both cases. This procedure simply doubles the amount of computational effort required for travel in one direction only. The second problem is solved by realizing that the minimum variable distance of 14 ft will produce the largest extreme stress for the two-hinged arch because of the nature of the stress influence lines, refer to the stress influence lines of Fig. 28. This simplifies the problem considerably since the distance between the two large axle loads is now fixed.

The extreme live load stress due to truck loading is found by determining the column which produces the extreme stress by an exhaustive search of ordinates of the stress influence line at the column points. One of the large axle loads is placed at the column point, and the remaining two axle loads are then placed in such a manner as to produce the largest extreme stress.

The magnitudes of the truck loads which are delivered to the arch rib are computed by the designer and submitted to the computer as input. P_{T2} will be equal to zero if the HS loading is not used. This procedure allows the designer to be more flexible in the selection of the truck loading.

The lane loading which is shown in Fig. 10c may be one of three different types as follows: (1) H10, (2) H15 or HS15 and (3) H20 or HS20, AASHO Specifications [64]. All three types consist of a uniformly distributed load plus a single roving concentrated load which may have one of two different magnitudes depending upon whether the resulting stress is primarily bending or primarily shear. The concentrated load for shear is proportioned as 13/9 of that for bending. The uniform lane load per 10 ft lane is as follows: H10 (0.32 kips/ft), HS15 (0.48 kips/ft), HS20 (0.64 kips/ft); and the concentrated load for bending is as follows: H10 (9 kips), HS15 (13.5 kips), HS20 (18 kips). The designer computes the portion of the uniform load (W_L) and the concentrated load for bending (P_L) which is supported by the arch rib. These loads are placed such that the extreme stresses are produced.

The idea of the stress influence line simplifies the computational effort required to find the extreme stresses due to lane load more than any other single concept. The roving concentrated load is located at one of the column points by making an exhaustive search of all possibilities, and the distributed load is distributed such that it covers only the desired area under the stress influence line. This area under the influence line is computed by the use of a modified trapezoid rule.

The sidewalk live load stress is computed in much the same manner as the stress for the lane load is computed, and it is computed within the same subprogram. This is possible because the sidewalk load is distributed in exactly the same manner as the lane loading. No additional influence line area computations are required. The designer computes the magnitude of the sidewalk load (W_s) which is delivered to the arch rib.

The longitudinal force (LF) is primarily due to the tractive force of a live load stopping on the bridge, refer to the AASHO Specifications [64]; however, the longitudinal force may also result from a temperature expansion or contraction of the bridge deck. In some cases this force will be directed toward the right for one lane of traffic and in the opposite direction for the other lane of traffic. Both directions have been considered in the computer program. The designer computes the magnitude of this load and submits it to the computer as input.

Preferably the longitudinal force should be taken by other bridge supports such as the abutments or piers, but it may be taken by the arch rib. If the force is taken by other bridge supports then LF = 0; however, if the force is taken by the arch rib, it is not zero and is divided equally into two parts as shown in Fig. 11a. The force may be placed at a constant elevation above the supports and carried to the arch rib through the bridge deck as in Fig. 11b, or it may be located at two column points and carried to the arch rib by means of bracing between columns as it is in Fig. 11c.

In order to compute the reaction components due to the longitudinal force (LF) the horizontal reaction of the right support B (Fig. 11a) is released. The internal moment (M) due to the longitudinal load is then computed for points along the arch rib at horizontal distances x and vertical distances y. A unit horizontal load is then placed at B producing the internal moment $y + \Delta Hx/S$ where ΔH = difference in elevation between supports and S = arch span. Using these two internal moments the horizontal reaction at support B is computed by

$$H_B = \frac{\displaystyle\int \frac{M(y + \frac{\Delta H}{S}x)}{EI}\, dL}{\displaystyle\int \frac{(y + \frac{\Delta H}{S}x)^2}{EI}\, dL} \tag{34}$$

where dL = incremental distance along the arch rib, E = modulus of elasticity and I = moment of inertia. The effect of thrust deflection is neglected for equation (34) since the internal forces resulting from the longitudinal force are normally small in comparison to dead load and live load internal forces. The horizontal reaction, H_A, can be found by

$$H_A = LF - H_B. \tag{35}$$

The resulting equations for H_A and H_B (equations 34 and 35) can be simplified considerably if the arch is symmetrical since in this case $H_A = H_B = 0.5\ (LF)$. The vertical support forces V_A and V_B are found by

$$V_A = V_B = \frac{1}{S}\left[\ (LF)\ a + H_B\ (\Delta H)\ \right] \tag{36}$$

where a = vertical distance between the left, high support and the line of action of the longitudinal force (LF).

For the most general case, the governing live load stress may be produced by any of three different types of live loads. These live loads were discussed in previous paragraphs and are interstate, truck and lane loading. Certainly for the final design all three types must be considered; however, it is usually possible to make all designs preceding the final few designs by considering only one type of live load. This is possible since one live load type usually controls the designs of all or most of the design points. The span lengths for the simple span beam at which each of these live loads begins to control is clearly defined, refer to the appendix of the AASHO Specifications [64]; however, the critical span lengths for the continuous span beam and the arch are variable. The author has attempted to estimate the range of span lengths at which each of the live loads controls. These estimates are as follows: (1) interstate loading, spans of less than 50 ft; (2) truck loading, spans between 100 ft and 400 ft; and (3) lane loading, spans over 500 ft. Defining the critical spans more clearly would be a possible project for future investigation. This could be done most easily for a symmetrical prismatic arch having a fixed number of column points and a fixed number of design points. The critical regions could then be plotted on a graph for various span lengths and height-to-span ratios.

Other Loads

Three other loads, which are considered in the solution of the arch, are those due to temperature change, wind and earthquake. All three of these loads will produce a significant amount of stress, and it is highly possible that they, in combination with the previous loads discussed, will produce the governing stress. Those loads which are not considered to act are earth pressure, buoyancy, centrifugal force, rib shortening due to creep and shrinkage, stream flow and ice pressure.

The horizontal reaction component (H_A or H_B) caused by a temperature change (t) is found according to Parcel [51], by

$$H_A = \frac{\gamma t S}{\int \frac{\left(y + \frac{\Delta H}{S} x\right)^2}{EI} dL} \tag{37}$$

where γ = coefficient of temperature expansion, S = span, ΔH = difference in support elevation, x = horizontal distance, y = vertical distance, dL = incremental arc length, E = modulus of elasticity and I = moment of inertia. As soon as the horizontal reaction is found then the vertical reaction and the internal forces can be found by statics.

The temperature change (t) may be either a temperature rise (TR) or a temperature drop (TD). Both conditions are considered, and the one which produces the governing stress is used. Normally the temperature drop is the most critical for two reasons: (1) if the structure is constructed under the average daytime temperature, the maximum temperature drop will be greater than the maximum temperature rise; and (2) a temperature drop produces compression in the top flange of the rib which is undesirable. A temperature rise produces tension in the top flange of the rib which produces a prestress to help carry the other loads. AASHO Specifications [64] require that a total change in temperature of $120^{\circ}F$ be used for steel structures in moderate climates. This total temperature change of $120^{\circ}F$ has been divided equally between a temperature rise ($TR = 60^{\circ}F$) and a temperature drop ($TD = 60^{\circ}F$) for all problems solved in this paper.

Wind pressure is applied to both the structure and to an assumed live load. The wind pressure on the structure (WIND) is equal to 75 psf, AASHO Specifications [64], and it is applied to the center of the exposed area. The wind on the live load (WL) is equal to 100 plf of bridge deck, and it is applied to an assumed live load at 6 ft above the deck surface according to AASHO Specifications [64]. Only 30 per cent of the wind force on the deck and live load is assumed to be carried by the arch ribs, and the remaining 70 percent is assumed to be carried by the abutments and piers. This procedure requires that the bridge deck be designed as a deep horizontal beam with a side or transverse loading of 70 per cent of the wind force. It is possible that this requirement should be 100 per cent of the wind load from the deck plus arch rib. If this approach were taken, then the arch rib would be required to carry no wind load at all.

Wind forces on the deck, columns and live load (Fig. 12) are assumed to be transferred to the arch rib by a vertical force (F_c) in each of the columns according to the approximate equation

$$F_c = \Sigma M_c / S_R \tag{38}$$

where M_c = moment caused by wind about point C at the base of the columns, and S_R = transverse distance between the two arch ribs. The wind force on the arch rib is assumed to be transferred to the arch by a vertical shearing force which is carried through the disphragms connecting the two arch ribs at the columns. This vertical shearing force is given by the approximate equation

$$F_c = \Sigma M_A / S_R \tag{39}$$

where M_A is the moment due to wind on the arch rib about a horizontal line through support A.

Although the wind may blow at any angle, wind in the transverse direction has been assumed to produce the maximum stress. The wind may either blow toward the bridge or away from it. In the one instance the column force would be compressive, and in the other the column force would be tension.

In regions where earthquakes may be anticipated, the lateral force produced by the earthquake (EQ) may be approximated according to the AASHO Specifications [64] by

$$EQ = (CEQ)\,D \tag{40}$$

where CEQ = earthquake coefficient which equals 0.02, 0.04 or 0.06 depending upon the foundation conditions, and D = dead load of the structure. The transverse force EQ is assumed to be transferred to the arch rib in much the same manner as the wind force, and the internal rib forces are computed by equations (38) and (39). Only 30 per cent of the force from the deck is assumed to be taken by the arch rib, and 70 per cent is assumed to be carried by the abutments and piers.

Stress Computations

Stress (σ) resulting from thrust (N) and moment (M) is computed by

$$\sigma = \frac{N}{A} + \frac{Mc}{I} \tag{41}$$

where A = area, I = moment of inertia and c = distance from the center of gravity to the extreme fiber. Compressive stress is assumed to be positive and tensile stress is negative. This equation is also used to compute the ordinates of the stress influence lines.

The procedure for finding the governing stress combinations and the corresponding allowable stress is discussed in the following paragraphs. Possible stress combinations are computed according to equations (42) to (48), and the allowable stress is computed according to equations (49) to (53). The stress calculations are computed for each design point, and the allowable stress is computed for each stress combination. The governing stress is then defined as that stress from one of the stress combinations which exceeds its particular allowable by the largest amount. With this procedure for selecting the governing stress, the load combination which produces the largest stress may not be selected as being the governing stress; however, the stress combination that is selected is expected to be the one which results in the largest flange thickness requirement.

The seven possible stress combinations to which each design point is subjected are listed in the following equations (42) to (48), taken from AASHO Specifications [64]; and the percentage by which each of these seven groups may exceed the allowable stress is listed to the right of the equations. The variables used in the equations imply the results or stress caused by loads which are as follows: D = deal load, LL = live load, I_m = impact coefficient, WIND = wind load, LF = longitudinal force, WL = wind on live load, TEMP = temperature and EQ = earthquake load.

Group I	= $D+LL(1 + I_m)$	0%	(42)
Group II	= D+WIND	25%	(43)
Group III	= Group I+LF+30%WIND+WL	25%	(44)
Group IV	= Group I+TEMP	25%	(45)
Group V	= Group II+TEMP	40%	(46)
Group VI	= Group III+TEMP	40%	(47)

Group VII = D+EQ 33-1/3% (48)

The allowable stress computations are made according to equations (49) to (53) which have been taken from the AASHO Specifications [64]. The basic allowable tensile stress due to bending (F_b) is found by

$$F_b = 0.55 F_y \tag{49}$$

where F_y = yield strength of the steel. The allowable bending stress (F_s) when both thrust and moment are present is

$$F_s = \frac{\frac{F_y}{\eta}}{1 + \left[0.25 + \frac{e_g c}{r^2}\right] \sec(\phi/2)} + \frac{Mc}{I} \tag{50}$$

where

$$\phi = \frac{0.75\ L_c}{r} \sqrt{\frac{\eta\left[F_s - \frac{Mc}{I}\right]}{E}} \tag{51}$$

M = moment, c = distance from the center of gravity of the section to the exterior fiber, η = strength constant, I = moment of inertia, e_g = largest eccentricity (moment divided by the thrust), r = radius of gyration, L_c = arc length between column points and E = modulus of elasticity. The allowable bending stress (F_r) as controlled by fatigue is given by

$$F_r = \frac{0.55\ F_y}{1 - \left[\frac{0.55\ F_y}{k_1 f_{ro}} - 1\right] R_f} \tag{52}$$

where

$$k_1 = 1.0 + \alpha \left[\frac{F_u}{58} - 1\right] \geq 1.0 , \tag{53}$$

f_{ro} = fatigue stress constant, R_f = ratio of minimum stress to maximum stress for fatigue stress calculations, α = constant for fatigue stress calculation and F_u = ultimate strength of the steel. The smallest of the stresses F_b, F_s or F_r as defined in equations (49) to (53) is the allowable.

Flange Thickness Adjustment

The flange thickness of the cross section may be adjusted in two different ways. The first method is to adjust the flange thickness (t_f) by stepping perhaps by 1/16 in. at a time, and this would be the best procedure if the flanges were limited to discrete thicknesses. The second procedure uses Newton's method for finding roots of a function, and this procedure is used in the present computer program. In this case, the new flange thickness (t_f) is found by writing the stress equation (41) as a function of flange thickness $f(t_f)$ where

$$f(t_f) = \frac{N}{A} + \frac{Mc}{I} - \sigma = 0. \tag{54}$$

The derivative $f'(t_f)$ of this equation (54) is taken with respect to the flange thickness (t_f) and substituted into Newton's iteration equation, written as

$$t_{f2} = t_{f1} - \frac{f(t_{f1})}{f'(t_{f1})} , \tag{55}$$

in order to find the new flange thickness. Iterations continue until the adjusted governing stress is within 10 psi of the allowable and the change in flange thickness adjustment is within 0.0001 in. In no case is the flange thickness permitted to become less than an arbitrary minimum of 1/2 in. or the AASHO Specification [64] minimum of

$$t_f = w \sqrt{550F_y/1.25} / 4000 \tag{56}$$

where w = flange width and F_y = yield stress.

Deflections

Deflection computations are made separately for dead and live loads (downward deflection positive), and these computations are made at each column point. This is accomplished through the use of deflection influence lines for each column point. Deflections are computed for the last few rib designs following the search for least cost assuming stress controls the design, and the deflections are computed by using influence lines in much the same manner as the stresses and forces are computed for the dead and live loads. The dead load deflection is used in order to adjust the elevation of the deck so that the deck will return to the proper elevation under dead load. According to the AASHO Specifications [64] the live load deflection is to be limited to 1/800 of the span where pedestrians do not commonly use the bridge and 1/1000 of the span when pedestrians frequently use the bridge. The Specifications further state that the live load deflection should be computed for the load which produces the maximum stress; however, this statement implies that the maximum deflection should be computed. With the use of the deflection influence lines both the maximum and minimum live load deflections are found.

The ordinate (δ_{ij}) of the deflection influence line at column point "i" due to a unit load at column point "j" may be computed, according to Parcel [51], by

$$\delta_{ij} = \int \left[\frac{m_i m_j}{EI} + \frac{n_i n_j}{AE} + \frac{m_i n_j}{AER} \right] dL \tag{57}$$

where m_i = moment caused by a unit dummy load at i, m_j = moment caused by a unit real load at j, n_i = thrust caused by a unit dummy load at i, n_j = thrust caused by a unit real load at j, E = modulus of elasticity, I = moment of inertia, A = section area, R = radius of curvature of the arc and dL = increment of arc length. Due to the fact that the third term in the deflection equation (57) is small, it has been neglected in the computer program; the third term has been included here only for completeness.

Final Few Designs and Output

During the final few designs some computations are made which are either unnecessary or were neglected in previous designs. These computations are: (1) all three types of live loads are considered; (2) the deflections due to both dead load and live load are computed; and (3) secondary stress due to deflection is accounted for.

The secondary stress due to deflection is caused by a change in the moment (ΔM) which is estimated by

$$\Delta M = H_A(\Delta_{LL} + \Delta_{DL}) \tag{58}$$

where H_A = ordinate of the horizontal reaction influence line, Δ_{DL} = deal load deflection and Δ_{LL} = maximum live load deflection. This change in moment is accounted for when computing the moment influence line of equation (29).

The internal shear forces due to all loads are computed for the final design. These forces are computed through the use of influence lines for shear and are computed in much the same manner as stress, moment, thrust and deflection. Equations (42) to (48) are used to find the governing shearing force, and in this instance the variables of the equations represent shearing force.

The amount of computer output has been kept to a minimum; however, all data necessary to complete the bridge design have been printed. These data include that necessary to compute the flange cut-off points, splice design, weld design, diaphragm design at column points and column design.

CHAPTER III

RESULTS, CONCLUSIONS AND RECOMMENDATIONS

Results Near Least Cost Design

The result of a typical design near the least cost design for the 310 ft parabolic arch which has H = 55 ft, ΔH = 2.48 ft, d = 70 in. and w = 35 in. (other input, Table 2) is given in Table 3. A sketch of this arch rib is shown in Fig. 30, and the flange thickness which is controlled by the section size requirements at either end of the element is shown.

Influence lines for the previous example are shown in Figs. 25 to 29. Along the abscissa, the units are in feet. Along the ordinate, moment, thrust and shear have no units. Stress has units of ksi/kip, and deflection has units of ft/kip. Moving and multiple loading conditions can be analyzed easily with the use of these influence lines. As an example of the use of the influence lines, assume a concentrated load of 100 kips is placed at column point no. 5. The results at point no. 5 are as follows: moment = 310 x 100 x 0.0544 = 1686.4 kip ft; thrust = 100 x 1.107 = 110.7 kips; shear= -100 x 0.526 = -52.6 kips; stress = 100 x 0.0388 = 3.88 ksi; and deflection = 100 x 0.000337 = 0.0337 ft.

Objective Function Surface

The objective function surface for the 310 ft parabolic arch (Fig. 13) was established by making many different designs for various values of arch height and web depth. The loads and other requirements which remained constant are given in Table 2. Most of the noise has been either eliminated or reduced to a small value. Web depth (d) has been taken as the abscissa, and the arch height (H) has been taken as the ordinate. The magnitude of the objective function (COST) has been indicated by contours of equal cost.

Several other parameters which are influenced by the designs for the objective function surface are shown in Figs. 14 to 23. These parameter surfaces include the following:

1. The total weight of the flanges and webs is shown in Fig. 14.

2. The approximate weight of the diaphragms and longitudinal stiffeners is shown in Fig. 15.

3. The total weight of the flanges, webs, diaphragms and longitudinal stiffeners is shown in Fig. 16. If the price of materials for flanges and webs equaled the price of materials for diaphragms and stiffeners, then the low point of the COST surface (Fig. 13) and the low point of the total weight surface (Fig. 16) would occur at the same values of arch height and web depth.

4. Maximum downward live load deflection contours are shown in Fig. 17. Deflection values used to produce this surface were found by selecting the largest downward live load deflection for each design. This surface indicates the values of arch height and web depth where live load deflection may become excessive; however, the deflections do not exceed 310 ft/800 = 0.388 ft (the allowable according to AASHO Specifications [64]) except in the upper left-hand corner.

5. Maximum upward live load deflection contours are shown in Fig. 18. Upward live load deflection contours are similar to downward deflection contours; however, downward live load deflections exceed upward live load deflections.

6. Maximum dead load deflection contours are shown in Fig. 19. This surface appears as a saddle with high values of deflection at very high and very low height-to-span ratios.

7. Maximum moment contours are shown in Fig. 20. The values for these contours were found by selecting the largest moment for each rib design. This surface is almost flat except at low values of height-to-span ratio and high values of web depth where divergence occurs.

8. Maximum thrust contours are shown in Fig. 21. This surface slopes gradually with increasing magnitudes at low height-to-span ratios.

9. Maximum shear contours are shown in Fig. 22. These contours are similar in shape to those for the COST surface of Fig. 13.

10. Maximum flange thickness contours are shown in Fig. 23. Flange thickness increases for low web depths due to strength requirements and for deep web depths due to minimum flange width-to-thickness ratios.

Factors Influencing the Least Cost Design

Items Influencing Cost

Although many items may influence the cost of an arch rib, only the more important items are considered herein. Two quantities which in some cases, may influence the cost of the arch rib considerably are flange width and flange thickness distribution. These two items are discussed in the following paragraphs.

Flange width (w) was varied for two cases in order to help indicate the influence of a flange width variation. In the first case, a series of designs for the 310 ft parabolic arch was selected with H = 55 ft, d = 70 in. and w = 10 to 70 in. The results are shown in Fig. 31 and in Table 4. The small increase in cost in Fig. 31 is due to the increased cost of diaphragms and flanges where they are controlled by minimum thickness dimensions. Also shown on the same graph is the actual maximum flange thickness and the minimum permitted flange thickness. Table 4 shows that the flange area requirement is almost a constant except at design points controlled by the minimum flange thickness. A second series of designs for the 310 ft parabolic arch was selected with H = 140 ft, d = 140 in. and w = 10 to 70 in. The results of this series of designs are shown in Fig. 32. For this case, the flange thickness is controlled by the minimum flange thickness dimensions, and the cost rises sharply with an increase in flange width.

From the discussion in the previous paragraph, the conclusion can be made that flange width variation becomes important when the flange thickness is controlled by the minimum thickness dimensions and where the flange thickness becomes excessively thick. Since excessively thick flanges are undesirable, they could be penalized by increasing the price of flange material when the flange thickness is larger than a certain amount. As noted in Table 4, if minimum flange thickness does not control, minor adjustments in the flange width may be made without making additional designs so long as the flange area is held constant with the width variation.

The flange thicknesses are independent variables that were made dependent by assuming that the governing stress equals the allowable stress at all design points, equation (5). Due to this assumption, the flange thickness is dependent upon the arch geometry, loads and material strength properties.

Other parameters and cost items which will influence either the cost of the arch rib alone or total cost of the arch bridge are as follows:

1. The shape of the rib section such as the box section (the one used), single web wide flange section or pipe section will influence the total cost of the arch rib.

2. Rib support conditions such as the three-hinged arch, the two-hinged arch (the one used) or the fixed-end arch will influence the cost of the arch rib, hinges, splices and abutments. Where favorable foundation conditions exist, the fixed-end arch is suspected to be the most economical for the multiple and moving loads of the arch bridge.

3. The strength of the steel used for the design will influence the cost of the arch rib.

4. Restricting the dimensions of the box section and plate thicknesses to discrete values will increase the cost.

5. The location and number of columns will influence the cost of the arch rib, the cost of the columns and the cost of the bridge deck. Considering the cost of the columns will influence the resulting height-to-span ratio.

6. The cost of the transverse weld across the flange can be used to decide whether to change flange thickness and weld or whether to continue a constant flange thickness through the design point and eliminate the weld.

7. The cost of the longitudinal welds will increase with increasing arc

length and increasing flange thickness; however, the total cost of the longitudinal welds may not vary sufficiently to influence the resulting design to any degree.

8. The cost of hinges and splices will become important when the three-hinged, two-hinged and fixed-end arch are compared.

9. The cost of the abutments will influence the choice of the resulting height-to-span ratio and the choice of support conditions. This is especially true if weak soil conditions are present.

Unimodality

The cost objective function was assumed to be unimodal during the search procedure. The objective function surface for the 310 ft parabolic arch (Fig. 13) is strongly unimodal; however, the following discussion is presented in order to lend strong support to the use of the unimodality assumption. Only positive values within the first quadrant are considered for the variation of the arch height and web depth since negative values have no physical meaning. Considering the cost objective function, equation (8), the following four arguments, one argument for each direction away from the low point, are made for unimodality.

1. With increasing arch height, the arc length (L) increases which increases the total cost. Divergence occurs because of geometric reasons.

2. With increasing web depth, the flange and web thicknesses are controlled by minimum thickness dimensions instead of stress, and the total cost increases as the section area increases. Divergence occurs because of geometric reasons.

3. With decreasing arch height, the internal forces and moments increase

requiring the section size to increase because of strength requirements, and the total cost increases as the section area increases. Divergence occurs because of strength reasons. This divergence at low height-to-span ratios is discussed in more detail under the topic Divergence.

4. With decreasing web depth, the flange area must increase to carry the load, and the flange area increases more than the decrease in web area producing an uneconomical design. As the web depth approaches zero the cost approaches a large finite value.

Smoothness

The objective function surface of the 310 ft parabolic arch (Fig. 13) appears to be smooth; however, small terraces may exist on the surface due to the fact that discrete element lengths are used between design points. Most of the lack of smoothness for the objective function curves of the 100 ft arches for the single cycle univariate search using the Fibonacci method (Fig. 42 and 43) may be due to noise, but some of the lack of smoothness is quite likely due to the fact that discrete plate thicknesses in addition to discrete element lengths are used. Plate thicknesses are permitted to vary continuously for the objective function surface of the 310 ft parabolic arch (Fig. 13) and for the multivariate searches. Doing this reduces the problems connected with the lack of smoothness. Using a continuous function to approximate for the weight of diaphragms and stiffeners aids in producing a smoother cost function for all designs. Other things which can be done to improve smoothness are to increase the number of design points and to permit flange thickness to vary between design points.

Noise

The process of converging to a stable design where the governing stress equals the allowable stress is shown in Figs. 33, 34 and 35. A series of designs each beginning with a different initial flange thickness and rib weight (initial values shown in Figs.) are made. Although noise quickly becomes small, within three to six redesigns, a small amount of noise still remains, in most cases, after nine to ten redesigns.

A problem of an entirely different nature is noted in Figs. 33, 34 and 35. All of the designs for each series of designs do not seem to converge to the same value. Assuming that the objective function is single valued may be incorrect. The difference between the designs at the end of the ninth or tenth redesign is 0.71 per cent for Fig. 33, 0.81 per cent for Fig. 34, and 0.45 per cent for Fig. 35. This percentage is small for these three cases; however, the difference may be larger for other arches. If multiple values of the objective function are present, the lowest value may be found if the flange thickness is reduced by a small amount at all design points and convergence is made again.

Noise will always be present except for arch ribs which are governed by minimum thickness dimensions where the arch rib is understressed and prismatic. Noise will be large if, during the search, the arch height or web depth is changed by a large amount as in the bracketing searches. Multiple values of the objective function may also be present. Noise and multiple values become considerably more important near the least cost design since the slope of the objective surface is small. For the 310 ft arch (Fig. 13) the slope within the center contour averages approximately 0.5 per cent per inch in the web depth direction.

Noise and multiple values of the objective function will cause less difficulty if the arch rib is forced to be prismatic. For this reason, the initial portion of the search for the least cost design should be made for a prismatic arch. Once the low point has been found for the prismatic arch, the final portion of the search can be made for a non-prismatic arch.

Divergence

Two types of divergence are of concern, (1) the divergence of the objective function surface and (2) the divergence of a particular design at low height-to-span ratios.

Divergence of the objective function surface occurs on three sides of the low point (increase in arch height, increase in web depth and decrease in arch height). With an increase in arch height and web depth the divergence is geometric while in the case of decreasing arch height the internal forces increase forcing the selection of a large section. The surface for maximum moment (Fig. 20), maximum thrust (Fig. 21) and maximum shear (Fig. 22) for the 310 ft parabolic arch indicate the extent of the divergence at low arch heights. Some of the results of reducing the height-to-span ratio are as follows: (1) The thrust due to all downward vertical loads increases. (2) The secondary effects of deflection increase the moment. (3) Temperature stresses increase.

Divergence with decreased arch height is shown in Fig. 36 where the total cost is plotted against the number of redesigns for three different arch height values. For the case when H = 20 ft, the cost diverges. For the case when

H = 30 ft, the cost is diverging, but it may eventually converge. For the case when H = 40 ft, the cost will most likely converge. A study was made to determine why divergence occurred, and the results are given in the following paragraphs.

Intermediate data were printed for the 310 ft parabolic arch having H = 20 ft and d = 20 in. The resulting stresses caused by the various loads are plotted for design point no. 5 in Fig. 37. All stresses except temperature stresses decrease as the redesigns require thicker flanges; however, the change is gradual with large changes in total cost. Convergence will probably never take place since the difference between the governing stress and the allowable stress is large. What is actually needed is a high strength steel with a higher allowable stress. The extent of the divergence is clearly obvious when the rib cost near the least cost design which is approximately $90, 000 is compared to the rib cost at the end of the ninth redesign which is $449, 000, a factor of approximately 5.

In order to estimate the influence of the rib weight, the arch is assumed to be parabolic, and the rib weight is assumed to be distributed uniformly along the arch span. The average weight (q) of the arch rib is equal to

$$q = A\,\rho\,\frac{L}{S} \tag{59}$$

where A = area, ρ= density, L = total arc length and S = arch span. Length of a parabolic arc can be approximated by the series

$$L = S\left[1 + \frac{8}{3}\left(\frac{H}{S}\right)^2 - \frac{32}{5}\left(\frac{H}{S}\right)^4 + \frac{256}{7}\left(\frac{H}{S}\right)^6 - \dots\right] \tag{60}$$

where H = height. Using this equation for length (L) the ratio L/S can be approximated by

$$L/S = 1 + \frac{8}{3}\left(\frac{H}{S}\right)^2 . \tag{61}$$

Stress (σ) due to dead load thrust at the support is given by

$$\sigma = \frac{qS}{8A}\sqrt{16 + \left(\frac{S}{H}\right)^2} . \tag{62}$$

The average dead load (q) is substituted into the equation for stress (σ) to give the approximate dead load stress as

$$\sigma = \frac{\rho S}{8}\left[1 + \frac{8}{3}\left(\frac{H}{S}\right)^2\right]\sqrt{16 + \left(\frac{S}{H}\right)^2} . \tag{63}$$

According to this equation (63) the dead load stress due to rib weight is not affected by a change in section area since area (A) does not appear in the equation; however, a change in height has a significant influence. The low point of the approximate equation (63) is found by calculus to be $H/S = \sqrt{3/32} = 0.307$.

Temperature stress increases for the series of redesigns of Fig. 37. Some approximations of how temperature influences the rib stress can be found by considering the change in stress at the crown of an arch whose moment of inertia (I) varies as the secant of the slope angle. The horizontal reaction component (H_A) for this arch due to a temperature change (t) is found by integrating equation (37) to give

$$H_A = 1.875\,\gamma\,t\,EI/H^2 \tag{64}$$

where γ = coefficient of temperature expansion, E = modulus of elasticity and H = height. Thrust (N) at the crown is

$$N = H_A\,, \tag{65}$$

and moment (M) at the crown is

$$M = (H_A)\,H. \tag{66}$$

These two results along with the area (A) of equation (21) and moment of inertia (I) of equation (22) are substituted into the equation (41) for stress (σ). The resulting equation for stress at the crown due to a temperature change is then given by

$$\sigma = 1.875\,\gamma t E \left[\frac{t_w d^3 + 3wt_f(d+t_f)^2 + wt_f^3}{12H^2(t_w d + wt_f)} + \frac{(d+2t_f)}{2H} \right] \tag{67}$$

where d = web depth, w = flange width, t_f = flange thickness and t_w = web thickness. According to equation (67) the temperature stress diverges for increasing t_f and decreasing H.

According to the discussion of the previous paragraphs, it may be concluded that temperature stresses increase with increased flange thickness; but dead load stresses due to rib weight do not. If temperature stress plus the stress from other loads exceeds the allowable by a large amount then the situation cannot be corrected by increasing the flange thickness. The best way of eliminating the problem of excessive stress is either to increase the arch height to where divergence does not occur or to use a steel with a higher allowable stress.

Fully Stressed

The two-hinged arch may not be fully stressed at the least cost design where a fully stressed design has been defined as a design for which the governing stress equals the allowable stress, equation (5). In spite of the fact that Gellatly [26], Venkayya [69] and Dwyer [21] warn that the fully stressed design may not be optimum, they encourage the use of the fully stressed design because of the following reasons. (1) The fully stressed design is an easy optimality condition to apply. (2) The fully stressed design usually produces rapid convergence to a stable design. (3) Any difference between the true optimum design and the fully stressed design is likely to be academic rather than practical. Razani [56] derives equations that can be used to determine whether the fully stressed condition is optimum or not. He uses a difficult mathematical procedure involving many computations in the proof. A more direct and simpler way (although not a proof) of checking to determine whether the fully stressed condition is optimum or not is as follows:

1. Find the fully stressed design with the noise reduced to a small amount.

2. Consider each of the elements between the design points as in Fig. 30. Add a small amount of material to one of the elements, and repeat the design until it has converged. If the total cost is reduced by adding material to one of the elements, then the fully stressed design is not optimal.

3. If an improved design is found then the design is saved, and a design which is better than this one is searched for. If an improved design is not found then the process of step 2 is continued by considering each element individually until all elements have been considered. If the total cost is reduced by adding material to any one of the elements then the fully stressed design is not optimal, but if the procedure fails to reduce the total cost then the fully stressed design is probably optimum.

Proportioning the Cross Section

The computer program searches to find the most economical web depth; however, some theoretical results are helpful in suggesting what the proportions of the section might be. Holt [35], among several other investigators [62, 4, 6, 50 and 61], gives optimum proportions for plate girders. The derivation in the following paragraph is made for prismatic elastic members having both moment (M) and thrust (N).

Assume that the cross sectional area (A) of the box section of Fig. 4 is given by

$$A = \frac{M_w d^2}{C_w} + 2A_F \tag{68}$$

where M_w = number of webs, d = web depth, $C_w = d/t_w$ = web depth-to-thickness ratio and A_F = flange area. The stress (σ) constraint equation (41), called "g" here, can be cast into the following form

$$g = 0 = \frac{N}{A} + \frac{Ne}{I/c} - \sigma \tag{69}$$

where $e = M/N$ = eccentricity, $c = s/2$ = approximate distance from the center of the section to the extreme fiber and the moment of inertia (I) is approximated by

$$I = \frac{M_w d^4}{12C_w} + \frac{A_F d^2}{2} \quad . \tag{70}$$

Using a Lagrange's multiplier (λ) and adding the constraint equation (g) to he area (A) produces the objective function ($\bar{A}$) as

$$\bar{A} = A + \lambda g \tag{71}$$

which has two independent variables (d, A_F) plus the Lagrange multiplier (λ). Taking derivatives of the objective function ($\bar{A}$) with respect to d, A_F and λ produces three equations which can be solved simultaneously to give the following three equations at optimum

$$2A_F = \frac{M_w D_o^2}{C_w} = \frac{A_o}{2} \quad , \tag{72}$$

$$A_o = \frac{2\,M_w D_o^2}{C_w} \quad , \tag{73}$$

and

$$D_o = \sqrt{\left[\frac{NC_w}{2M_w\sigma}\right]\left[1 + \frac{3e}{D_o}\right]} \tag{74}$$

where D_o = optimum section depth and A_o = optimum section area. Equation (72) shows that the total flange area must be equal to the total web area and one-half of the total area at optimum. Equation (74) for optimum depth (D_o) can be solved by an iterative approach.

Some approximations can be made using the equations (72), (73) and (74) even though the arch rib is not prismatic as required and the arch rib has diaphragms and longitudinal stiffeners. Both effects tend to reduce the optimum web depth and increase the optimum area. Governing moments, governing thrusts and allowable stresses for the 310 ft parabolic arch (H = 55 ft, d = 70 in.) (Table 3) are used to predict the section properties. Results of these predictions are given in Table 5. The actual web depth is 85 per cent of the average predicted web depth of 82.14 in., and the actual average area is 104.6 per cent of the average predicted area. When the total flange volume (results of Table 3) is divided by the total rib volume, the ratio is 0.694 as compared to 0.5 which is predicted by equation (72) for members having no web stiffening elements and being prismatic.

Acceleration, Deceleration and Step Size

Accelerating the step size by one-half of the initial step size with success and decelerating the step size by one-half of the present step size with failure seems to be a good procedure. On the other hand, even though a small step size can be increased with success, the designer is cautioned against selecting a small initial step size. If a very small initial step size is used, the noise may be large enough to keep the search essentially fixed; and the global least cost design will not be approached. An initial step size of 5.0 (5.0 ft for arch height direction and 5.0 in. for web depth direction) and a tolerance of 2.0 (2.0 ft for arch height variation and 2.0 in. for web depth variation) seems to be satisfactory for the

310 ft arch. An initial step size of 18.0 and a tolerance of 6.0 seems to be satisfactory for the 600 ft arch. If these two values are read into the computer as zero, the computer selects an initial step size of 0.03 S and a tolerance of 0.01 S where S = span length in feet. Using a large initial step size and reducing it gradually will increase the chances that the search will end within the region of the least cost design.

Convergence of the Search

Converging to the least cost design and stopping the search is one of the most important stages of the search procedure. The low point that is found must be checked since noise will often lead the search to an incorrect solution. For the arch, good results seem to be produced (1) if a large initial step size is started with and reduced gradually; (2) if convergence requirements for noise are made more stringent as the least cost design is approached; and (3) if a previously found low point is checked a second time before it is used.

Preselecting Governing Constraints

Some preselection of governing constraints and loading conditions is made for the two-hinged arch. This procedure seems to be quite successful, and it is a good way of reducing the required computer time. For the arch, the following things are done in order to reduce computer time. (1) Depending upon the designer's discretion, he can select only one live load instead of using all three for the preliminary designs. (2) The secondary effects of deflections are not considered until the last few designs; therefore, deflection calculations are not required for most of the designs leading to the least cost design. (3) Live load deflections are assumed not to be excessive, and they are checked only after the least cost design is found assuming stress controls the design. (4) Initially noise is not eliminated, but noise reduction becomes more stringent as the least cost design is approached. (5) Shear forces and dead load deflections are computed only for the final design. Additional things that could be done are as follows: (1) Preliminary designs could be made considering only loading groups I and III instead of using all seven. (2) Many of the preliminary designs could be made for a prismatic arch. (3) Estimates of the optimum section proportions could be made using theoretical equations. (4) Some of the preliminary designs could be made using an approximate allowable stress.

Deflections

Reducing Excessive Deflections

Excessive live load deflections are reduced in various ways during the development of the computer program. When the univariate search using the Fibonacci procedure is used, deflections are reduced by increasing the web depth and/or increasing the flange thickness. When the multivariate searches such as the gradient, conjugate direction and univariate with the stepping procedure are used, live load deflections are reduced by restricting the search to the portion of the objective function surface where deflections are not excessive. If the live load deflections are limited to 0.15 ft then the restricted objective function surface appears as in Fig. 24. This restricted surface is arrived at by comparing the objective function surface of Fig. 13 and the downward live load deflections of Fig. 17.

Examples of Stress and Deflection Constraint Relationships

Restricting the search to the portion of the objective function surface for which stress controls the design rather than deflection ignores the possibility that deflections for some rare instances may be reduced more efficiently by reducing the stress level. The following discussion shows that the low point for some other structures does not necessarily occur on the boundary of the restricted surface where the design is controlled by stress.

Consider an arch made of two straight segments with a load in the center. If the allowable stress (σ) governs the design then the cost of the structure (COST) is

$$\text{COST} = \frac{pQS}{\sigma}\left[\frac{H}{S} + \frac{S}{4H}\right] \tag{75}$$

where p = price, Q = load, S = span and H = height. If the deflection (Δ) governs the design then the cost is

$$\text{COST} = \frac{pQS^2}{\Delta E}\left[\left(\frac{H}{S}\right)^2 + \frac{1}{2} + \left(\frac{S}{4H}\right)^2\right] \tag{76}$$

where E = modulus of elasticity. The optimum H/S ratio in both cases is found by calculus to be one-half.

A typical problem showing the relationship of these two curves is shown in Fig. 38. The curves for this graph assume that S = 20 ft, Q = 20 kips, σ = 20 ksi, p = \$0.20/lb and E = 30000 ksi. For this example, the optimum design is controlled by either deflection or stress and never by both simultaneously. Moving up along the stress constraint curve to correct for excessive deflections will not produce optimum.

As a second example, consider an elastic prismatic beam constrained by stress (σ) and/or deflection (Δ). The constraint equation (41) for stress is simplified by setting thrust (N) equal to zero to arrive at

$$\sigma = \frac{Mc}{I} \tag{77}$$

where M = moment, c = distance from the center of gravity to the extreme fiber which is approximately one-half of the web depth (d/2), and I = moment of inertia as defined by the approximate equation (70). The constraint equation for deflection is

$$\Delta = \frac{K}{I} \tag{78}$$

where K = stiffness constant.

With these two constraint equations and the equation (68) for area (A), curves can be drawn as in Fig. 39. Assume a 40 ft simple span beam supporting a load of 1.0 kip/ft which includes the beam weight. Let the allowable stress σ = 24 ksi; the number of webs M_w = 1; and the web depth-to-thickness ration C_w = 69.

Observing Fig. 39, at optimum the ratio of flange area to section area ($2A_f/A$) is one-half if stress governs and one-fourth if deflection governs. If the limiting deflection is 1.5 in. or 1.7 in. then the optimum can be found by moving up along the stress constraint curve to the intersection or boundary where both stress and deflection control simultaneously. If the limiting deflection is 1.3 in. then the optimum cannot be found by moving up along the stress constraint curve to the intersection point since the low point occurs on the deflection constraint curve away from the intersection point. If the deflection is limited to 1.3 in. or less then the design is governed by the deflection constraint curve. This beam example is probably similar to the arch rib if only bending deflections are considered. Excessive deflections can be corrected to a certain extent by increasing web depth and reducing flange area to keep a design which is governed by stress; however, beyond a certain point both web depth and flange area must be increased to produce an under-stressed design. For the arch which is non-prismatic and has web stiffening elements, the ratio of the flange volume to the total volume is expected to be approximately 0.30 to 0.35 if the correction for excessive deflection is large. As noted in Fig. 39, a small movement from the true optimum ratio will not produce large changes in section size requirements.

As a third example, consider the least cost design of a constant height bar joist by Harriman [31]. A 30 ft simple span bar joist loaded with a uniform load of 0.54 kips/ft is designed according to the AISC specifications [1] using A36 steel. The cost items are as follows: steel cost at $0.30 per pound, cost of fabrication at each panel point at $4.00 and cost of increasing the building height in order to house a deeper joist at $100 per ft. The cost contours of the objective surface of Fig. 40 show four levels of maximum deflection limitation ($\Delta = \infty$, 1.6 in., 1.0 in., 0.5 in.) where one portion of the surface is controlled entirely by stress and the other portion is controlled entirely by the maximum deflection limitation. Consider the four cases of Fig. 40 as follows: (1) when the deflection is permitted to be large ($\Delta = \infty$), stress limits the design for the entire surface, and all designs are governed by the allowable stress. (2) When deflection is limited to 1.6 in., the lower portion of the objective surface is controlled by the maximum deflection limitation producing designs which are under stressed. The least cost design, however, appears to occur on the portion of the surface controlled entirely by the allowable stress. For this case, the least cost design is governed by stress, but the actual maximum deflection at the least cost design is quite likely less than the limiting deflection of 1.6 in. (3) When deflection is limited to 1.0 in., the least cost design occurs on the boundary where the two surfaces intersect forming a cusp. The least cost design is still governed by stress, and in this case, the actual maximum deflection is exactly equal to the limiting deflection of 1.0 in. (4) When the deflection is limited to 0.5 in., the least cost design occurs on the portion of the surface controlled entirely by deflection. The actual maximum deflection is exactly equal the limiting deflection of 0.5 in., and the least cost design is not governed by stress but by deflection.

Arch Objective Surface for a Deflection Constraint

Finding the new raised surface to correct for excessive deflections by increasing flange thickness can be done; however, the amount of computational effort required will be large. The following discussion gives some theory related to finding the new surface and suggests some practical means of approximating it.

Prager [54] states that the necessary and sufficient optimality condition which must be used in order to reduce the maximum deflection for a single loading condition is that strain energy density be a constant, i.e. the structure must be uniformly strained. For multiple loading conditions, Venkayya [69] approximates the optimality condition by stating that the largest average strain energy density, not necessarily caused by the same load, is constant throughout the structure.

For the arch, maximum deflections can be reduced by increasing the flange thickness of some or all of the elements between the design points. The element which should be changed first can be found by increasing the flange thickness of each element in succession to determine which change produces the best results. Flange thickness is added to this element, and then each of the elements is changed a second time. The cycle is repeated adding small amounts of flange thickness to the appropriate element until the maximum live load deflection is equal to or less than the allowable.

Since searching for the element which should be increased first is time consuming, an approximate means of reducing deflection is more practical. One way of doing this is to increase the flange thickness of the most severely strained elements first, and the procedure is continued seeking a uniformly strained condition. A second method is to increase the flange thickness of all elements by a percentage or by an incremental amount. A third method is to increase the flange thickness by reducing the allowable stress. The third method was used for the univariate search using the Fibonacci method. If the ratio of total flange volume to total section volume is less than or equal to approximately 0.3, a fourth method is recommended, and that is to increase both web depth and flange thickness simultaneously keeping the ratio approximately 0.3.

Recommended Procedure for Reducing Deflections

Reducing excessive deflection by restricting the stress objective surface as in Fig. 24 may be incorrect when the adjustment for excessive deflection is large. This fact was explained when considering the three examples of Figs. 38, 39 and 40. On the other hand, searching for the surface controlled by deflection by increasing the flange thickness will be time consuming, and it is to be avoided if possible.

When live load deflection is excessive, the deflection may be reduced in the following manner. Adjust the web depth and arch height to the boundary where deflections first become excessive keeping a design where the flange thicknesses are governed by stress, but do not reduce the ratio of total flange volume to total section volume to less than approximately 0.3. In cases where the deflection is still excessive, gradually increase both flange volume and web volume simultaneously for continuously varying plate thicknesses keeping the ratio approximately 0.3.

Discussion of Search Methods

Exhaustive Search

The exhaustive search is used to find the governing loading condition, and for this purpose, the exhaustive search produces good results. On the other hand, if continuity does exist in the bridge deck and the columns where hinges are now, the influence lines will be curved instead of being made of straight segments; and the exhaustive search for finding the governing live load results may not be possible.

Single Cycle Univariate Search

The single cycle univariate search is used for the parabolic, circular and straight segmented arches having discrete web and flange thickness values. Variation of the arch geometry (height for parabolic and circular arches and elevation of design points for the straight segmented arch) is considered as one part of the cycle in the search procedure, and variation of web depth is considered as the second part of the cycle. If deflections are excessive, they are reduced by either increasing the web depth or flange thickness by making equal increment steps. Important input data for all cases is given in Table 2.

The results for a 100 ft parabolic and circular arch are shown in Figs. 41, 42 and 43. In Fig. 41, the equal interval search is used, and in Figs. 42 and 43, the Fibonacci search is used. Fig. 44 shows the Fibonacci search for the 310 ft parabolic arch, and Fig. 45 shows the Fibonacci search for the 600 ft parabolic arch. Fig. 46 shows the Fibonacci search for the 100 ft straight segmented arch. For the 100 ft straight segmented arch, the elevation of each design point is varied in succession, and the cycle is continued until all design points are within a given tolerance. Only the last of four cycles is shown in Fig. 46. After the geometry is found for the straight segmented arch, the web depth is varied to find the resulting web depth.

As noted in Figs. 41 to 46 the lack of smoothness is a problem. For the 100 ft arches, the lack of smoothness is more severe than for longer spans. The use of discrete web and flange thicknesses and noise produce most of this irregularity. For later multivariate searches, the thicknesses are permitted to vary continuously. Noise is reduced for the Fibonacci searches by repeating each design five times at each point, and by adjusting the flange thickness between large web depth changes. Because noise is more extreme for the bracketing searches,they are not recommended for the two-hinged arch design.

Fig. 47 summarizes the results of a series of designs between 100 ft to 600 ft for arch ribs having discrete plate thicknesses, and Table 6 gives the resulting H/S and d/S ratios for these designs. Both the parabolic and circular arch designs result from the single cycle univariate search using the Fibonacci procedure. Because only one cycle is used and because noise is present, convergence to the least cost design may not be complete; however, all designs are expected to be within the general region of the global least cost design. For this particular graph (Fig. 47) the least cost design for the straight segmented arches has not been searched for, but the elevation of the design points of these straight segmented arch designs are placed on a circular pattern for the geometry of the resulting circular arch. For all designs, the circular arch is more economical than the parabolic arch, and the straight segmented arch is more economical than the circular arch.

Multivariate Searches

Four multivariate searches are studied, and they are gradient, conjugate direction, univariate and ravine searches. All of these searches use stepping once the direction is established. The univariate search with stepping produces the best results in the presence of noise, and for this reason, it is recommended for adjusting the arch height and web depth for the objective surface controlled by stress and for the objective surface restricted by excessive deflections.

Figs. 48, 49 and 50 show three univariate searches for the least cost 310 ft parabolic arch. The improved steps of the search path in Fig. 48 are given in Table 7. As explained previously, these steps may not always be improvements over previous steps because of noise. Fig. 51 shows two univariate searches for the least cost 600 ft parabolic arch. The objective surface for the 600 ft parabolic arch is not established.

Recommended Design Procedure

The following will be a good procedure to follow in order to search for the least cost design.

1. Using continuously varying plate thickness values, begin the search as nearly as possible to the suspected least cost design, perhaps $H = 0.17S$ and $d = 0.02S$ where H = height, d = web depth and S = span. The resulting web depth will vary, considerably more with increasing span lengths than the resulting arch height, refer to Table 6.

2. Simplify some preliminary designs by doing the following: Limit the loading combinations. Consider only a prismatic section. Eliminate deflection calculations until the arch has been adjusted for stress considerations. Use an approximate allowable stress.

3. Use the univariate search with the stepping procedure, and accelerate and decelerate the step size with success and failure of the step. Adjust the arch height for a prismatic arch, and approximate the web depth proportions by using theoretical equations.

4. Once the approximate web depth has been found for a prismatic arch, use a nonprismatic arch and the correct allowable stresses. Continue to use the univariate search with the stepping procedure, and reduce the basic step size gradually with each direction change according to the harmonic series (1, 1/2, 1/3, 1/4, ...) until the step size is within the tolerance. Also make the reduction of noise more stringent as the least cost design is approached.

5. Once the search is completed assuming stress controls the design and using continuously varying plate thicknesses do the following: Consider all loading combinations. Adjust flange width seeking reduced rib costs, and at the same time, penalize thick flanges. Compute deflections, and consider secondary effects of deflections. Use discrete plate thicknesses. Find the appropriate flange cut-off points, and compare the cost of extending the constant flange thickness to the next cut-off point versus the cost of making the transverse weld. Adjust arch height and web depth again if large changes in total cost are produced by making any of these additions. If live load deflections are not excessive, proceed with the final design; however, if live load deflections are excessive, proceed to the next step.

6. For excessive live load deflections, adjust web depth and arch height until deflections are either satisfied or until the ratio of the total flange volume

to total section volume approaches 0.3. At this stage, increase both web depth and flange thickness alternately or simultaneously keeping the ratio approximately 0.3 seeking a uniformly strained condition. Continue this procedure until deflections are satisfactory.

7. For the final design do the following: Repeat the final design several times in order to eliminate noise. Reduce the flange thicknesses, and converge again to check for multiple values of the objective function. Compute final values such as the governing shear forces and column loads which were previously omitted.

Conclusions

The following conclusions may be made from the study of the search techniques for the design of two-hinged highway arch ribs.

1. The cost objective function for the two-hinged highway arch rib considering a stress limit and having arch height and web depth as the independent variables is strongly unimodal within the region of practical designs for the 310 ft parabolic arch.

2. Convergence of a particular design for fixed values of arch height and web depth to a stable value, defined as noise, and possible multiple values of the objective function are the most critical problems relating to the least cost design of the two-hinged, non-prismatic arch.

3. The magnitude of noise will be large with large changes in arch height and web depth.

4. Due to the presence of noise the equal interval and stepping search are superior to the golden section and Fibonacci search, and the univariate search is superior to the conjugate direction, gradient and ravine searches.

5. Divergence of the cost objective function at low height-to-span ratios is due to two main causes: (a) internal forces and moments increase with decreasing arch height requiring a larger section size, and (b) temperature stress diverges with decreasing arch height and increasing flange thickness.

6. Live load deflections are not excessive for any of the arches investigated.

7. Excessive live load deflections can be reduced by adjusting web depth and arch height keeping a design where the flange thicknesses are governed by stress; however, if the correction for excessive live load deflection is large then increasing the flange thickness in addition to adjusting the web depth and arch height may be more economical.

Recommendations

The following recommendations are made for future investigations.

1. Use the recommended design procedure discussed previously.

2. Extend the application of the computer program in the following ways: Consider additional support conditions such as the three-hinged arch and the fixed-end arch. Consider more cost and design items such as the transverse weld, the columns, the abutments, the splices and hinges.

3. Find the height-to-span and depth-to-span ratios more precisely for the least cost design of typical arches.

4. Consider the application of the techniques discussed by Jones and Hague [38] using a penalty function to solve equations (2) and (3).

APPENDICES

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APPENDIX A

Tables

Table 1. Percentage Reduction in H_A When Thrust Deflections are Considered for Equation (27)

H/S	Percentage Reduction	
0.05	8.11%	$k_c = 0.50$
0.10	2.16	$S = 100$ ft
0.15	0.97	$A = 70$ in.^{2}
0.20	0.55	$I = 11,863$ in.^{4}
0.25	0.35	
0.30	0.26	

Table 2. Selected Input Data (All Arches)

PI	= 65.64 kips	FY	= 36 ksi
PT1	= 87.52 kips	FU	= 58 ksi
PT2	= 87.52 kips	ETA	= 1.80
PL	= 49.23 kips	CEQ	= 0
WL	= 1.7504 kips/ft	PRICE1	= \$0.25 per lb
WS	= 0 kips/ft	PRICE2	= \$0.35 per lb
LONGF	= 0 kips	TOLDEFL	= 0.00125 ft/ft
TR	= 60^{o}F		
TD	= 60^{o}F		

DELH	= 0 (except 310 ft, DELH = 2.48 ft)
HORIZD	= 27.33 ft
DECKD	= 6.50 ft
HLF	= 0 ft
L1	= 0
L2	= 0

SPAN ft	HDECK ft	WDECK kips/ft	FC kips	WC kips/ft
100	35	5.5	60	.04
200	70	5.5	60	.08
300	105	5.6	90	.12
310	90.65	5.584	88	.118
400	140	5.8	130	.16
500	175	6.1	170	.20
600	210	6.3	210	.24

Distance to Columns

(XC = X except $X_0 = 0$, X_{NP} = SPAN)

Design Point NC=NP	100 ft Arch	200 ft	300 ft	310 ft	400 ft	500 ft	600 ft
0	0	-10	-15	-10	-20	-25	-30
1	20	10	15	20	20	25	30
2	40	30	45	50	60	75	90
3	60	50	75	80	100	125	150
4	80	70	105	110	140	175	210
5	100	90	135	140	180	225	270
6		110	165	170	220	275	330
7		130	195	200	260	325	390
8		150	225	230	300	375	450
9		170	255	260	340	425	510
10		190	285	290	380	475	570
11		210	315	320	420	525	630

Table 3. Selected Output (310 ft Parabolic Arch)

HEIGHT	= 55 ft	RIBWGT	= 323638 lb
DELH	= 2.48 ft	STIFFENERS	= 27070 lb
DEPTH	= 70 in.	COST	= $90384
WIDTH	= 35 in.		
TWEB	= 0.6118 in.	FCLL	= 163 kips

Design Point NP	TFLG in.	CRITSTR ksi	ALLOWABLE ksi	CRITM kip ft
0	1.1012	10.3	11.9	0
1	2.1750	11.9	11.9	3426
2	3.1189	11.5	11.5	5340
3	3.0512	11.5	11.5	5306
4	2.5612	11.5	11.5	4290
5	2.0417	11.7	11.7	3326
6	2.0428	11.7	11.7	3325
7	2.5639	11.5	11.5	4291
8	3.0543	11.5	11.5	5312
9	3.1257	11.5	11.5	5349
10	2.1833	11.9	11.9	3433
11	1.1012	10.4	11.9	0

CRITN kips	CRITV kips	DLD ft	LMAXD ft	LMIND ft
1674	140	.0000	.0000	.0000
1262	184	.0073	.0712	-.0634
1217	158	.0155	.1448	-.1308
1168	160	.0213	.1667	-.1503
1156	179	.0249	.1333	-.1204
1146	189	.0268	.0728	-.0570
1149	189	.0269	.0725	-.0567
1162	179	.0254	.1332	-.1203
1176	160	.0219	.1668	-.1504
1226	158	.0161	.1451	-.1311
1273	183	.0076	.0714	-.0635
1687	138	.0000	.0000	.0000

Table 4. Flange Area for Flange Width Variation (Fig. 31)

Flange Width (in.), Area in sq in.

Design Point NP	10	14	20	25	30	35	40	50	70
0	9.91	9.98	12.59	19.67	28.32	38.54	50.34	78.66	154.18
1	75.59	79.78	75.29	75.18	75.09	75.04	75.00	78.66	154.18
2	108.91	109.30	109.26	109.19	109.11	109.05	109.00	108.81	154.18
3	107.08	107.41	107.35	107.27	107.19	107.12	107.07	106.81	154.18
4	90.13	90.28	90.18	90.11	90.04	90.00	89.96	89.95	154.18
5	69.64	69.68	69.68	69.68	69.68	69.67	69.68	78.66	154.18
6	69.79	69.83	69.84	69.83	69.81	69.81	69.82	78.66	154.18
7	90.44	90.59	90.51	90.43	90.34	90.29	90.25	90.24	154.18
8	107.52	107.85	107.81	107.73	107.62	107.55	107.50	107.23	154.18
9	109.40	109.80	109.79	109.73	109.60	109.54	109.50	109.30	154.18
10	76.03	80.26	80.11	80.02	75.53	75.48	75.44	78.66	154.18
11	10.31	10.39	12.59	19.67	28.32	38.54	50.34	78.66	154.18

Table 5. Predicted Section Depth Proportions

Using Results of Table 3

Design Point NP	Actual Area $in.^2$	Predicted Depth Equation (74) in.	Predicted Area Equation (73) $in.^2$
0	163	63.43	141
1	238	81.64	233
2	304	91.00	290
3	299	90.35	285
4	265	85.76	257
5	229	80.29	225
6	229	80.32	226
7	265	85.83	256
8	299	90.46	286
9	304	91.13	290
10	238	81.80	234
11	163	63.68	142
Ave	250	82.14	239

Actual Ave. Area = 104.6% Ave. Predicted Area

Actual Depth = 85% Ave. Predicted Depth

$d/t_w = 14400/\sqrt{550F_Y/1.25}$

Table 6. H/S and d/S Ratios for Designs for Fig. 47

SPAN	CIRCULAR		PARABOLIC	
	H/S	d/S	H/S	d/S
100 ft.	.171	.0330	.171	.0400
200	.190	.0232	.185	.0235
300	.172	.0195	.172	.0195
400	.180	.0163	.185	.0173
500	.177	.0163	.199	.0163
600	.150	.0165	.185	.0165

Table 7. Improved Search Steps, Fig. 48

No.	COST	DEPTH	HEIGHT
1	$249630	140.0 in.	90.0 ft
2	216600	130.7	90.0
3	172680	116.8	90.0
4	122540	98.2	90.0
5	95180	74.9	90.0
6	94480	67.9	90.0
7	92390	67.9	80.7
8	90850	67.9	66.8
9	90290	67.9	62.1
10	90390	67.9	55.1
11	90250	71.0	55.1
12	90240	75.7	55.1
13	90190	72.6	55.1
14	90130	72.6	58.2
15	90060	72.6	60.6
16	90180	69.5	60.6
17	90180	71.8	60.6
18	89950	73.5	60.6
19	90124	71.68	60.55

APPENDIX B

Figures

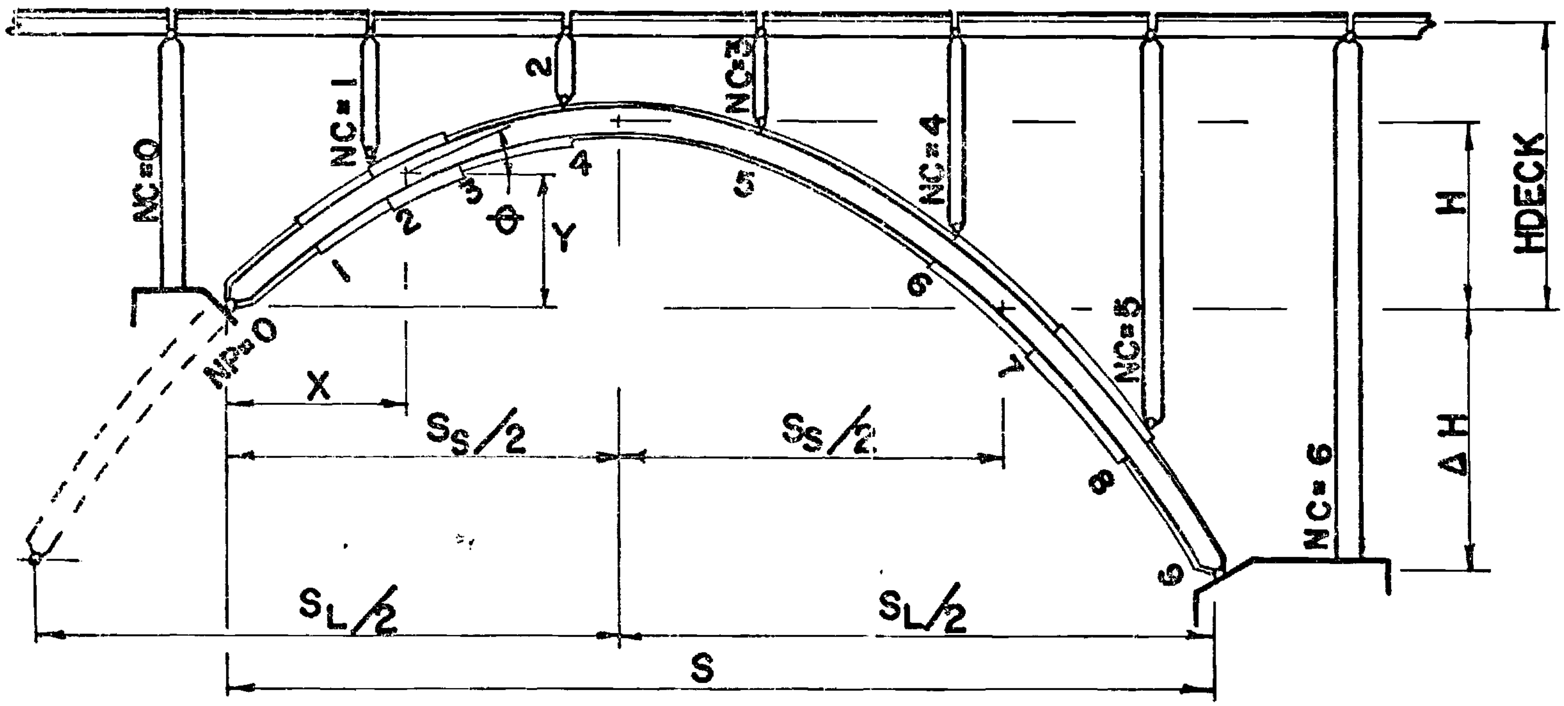


Fig. 1. Unsymmetrical Parabolic Arch.

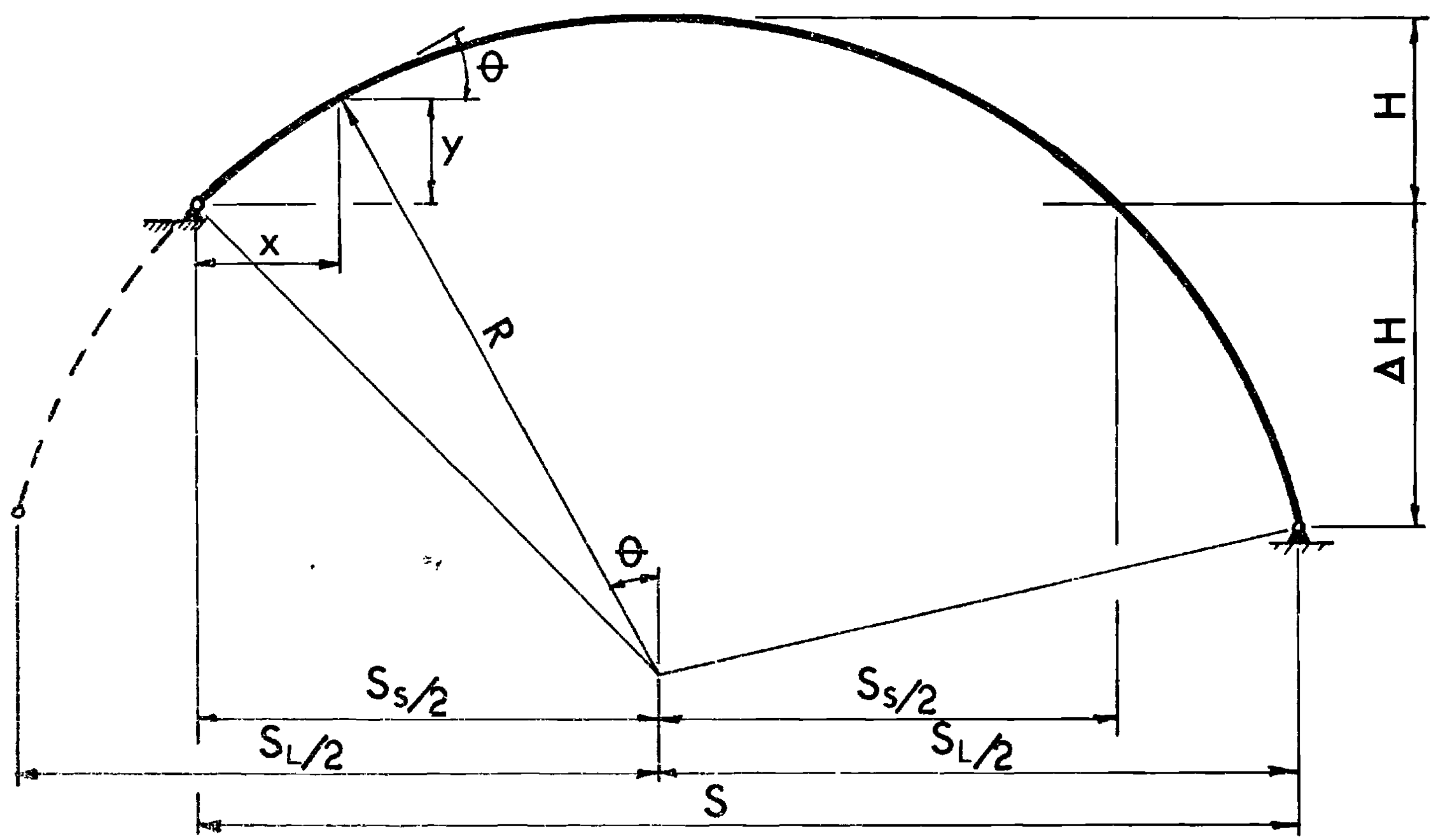


Fig. 2. Unsymmetrical Circular Arch.

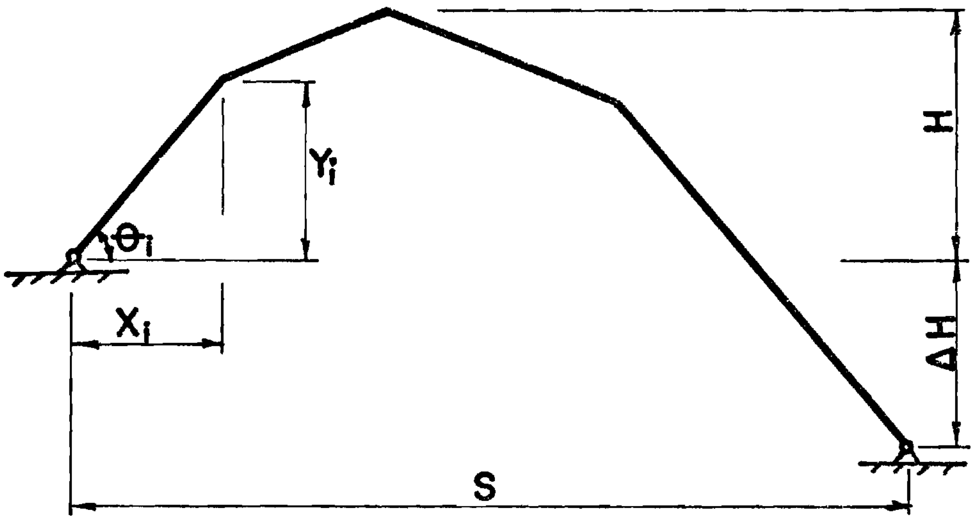


Fig. 3. Straight Segmented Arch.

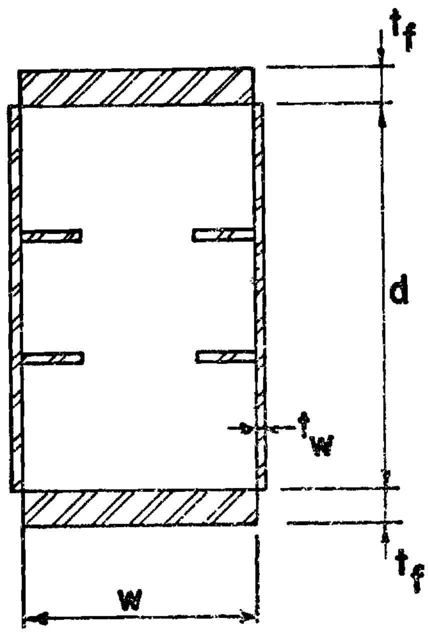


Fig. 4. Arch Rib Cross Section.

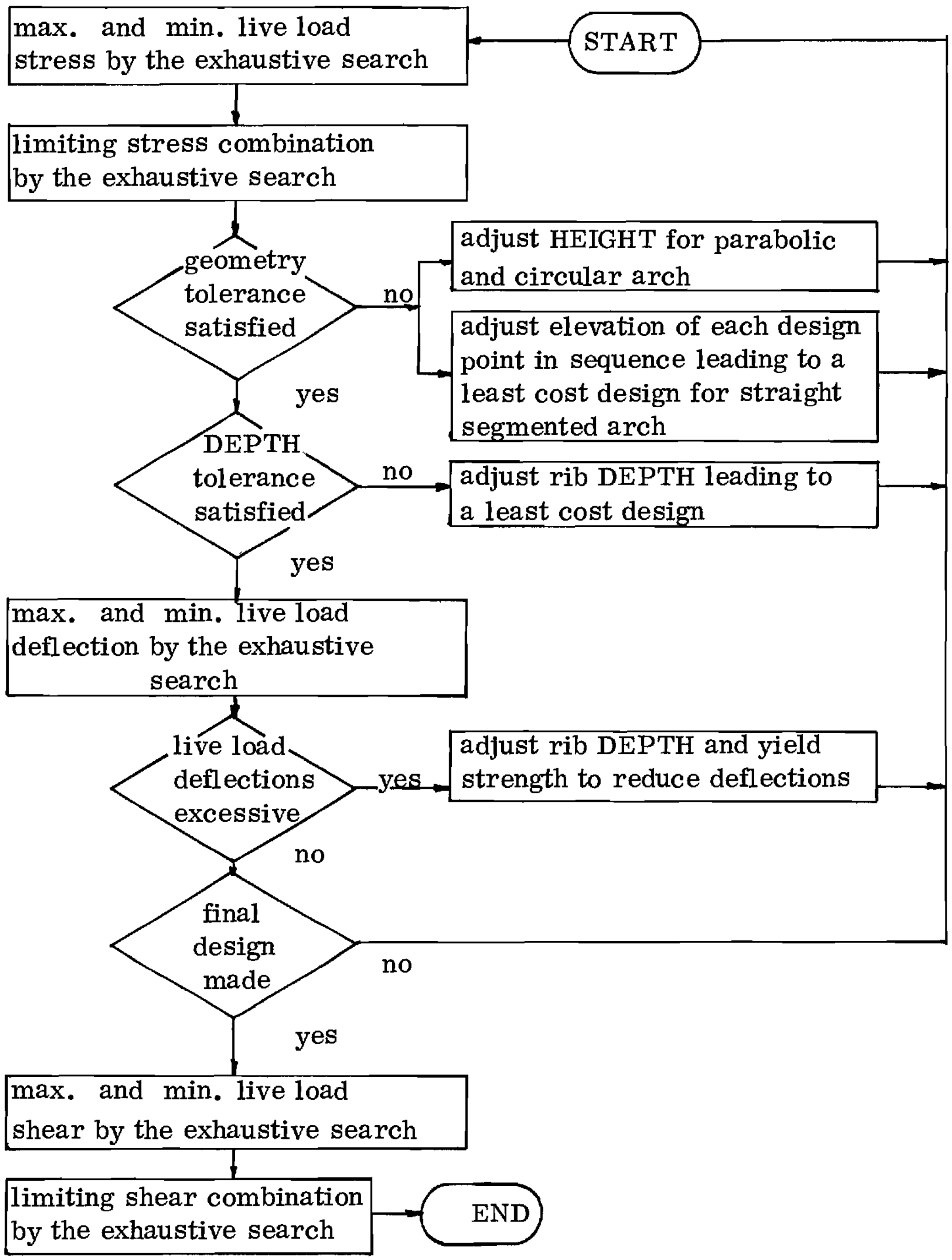


Fig. 5. Search Plan Using the Single Cycle Univariate Search

with the Fibonacci Procedure.

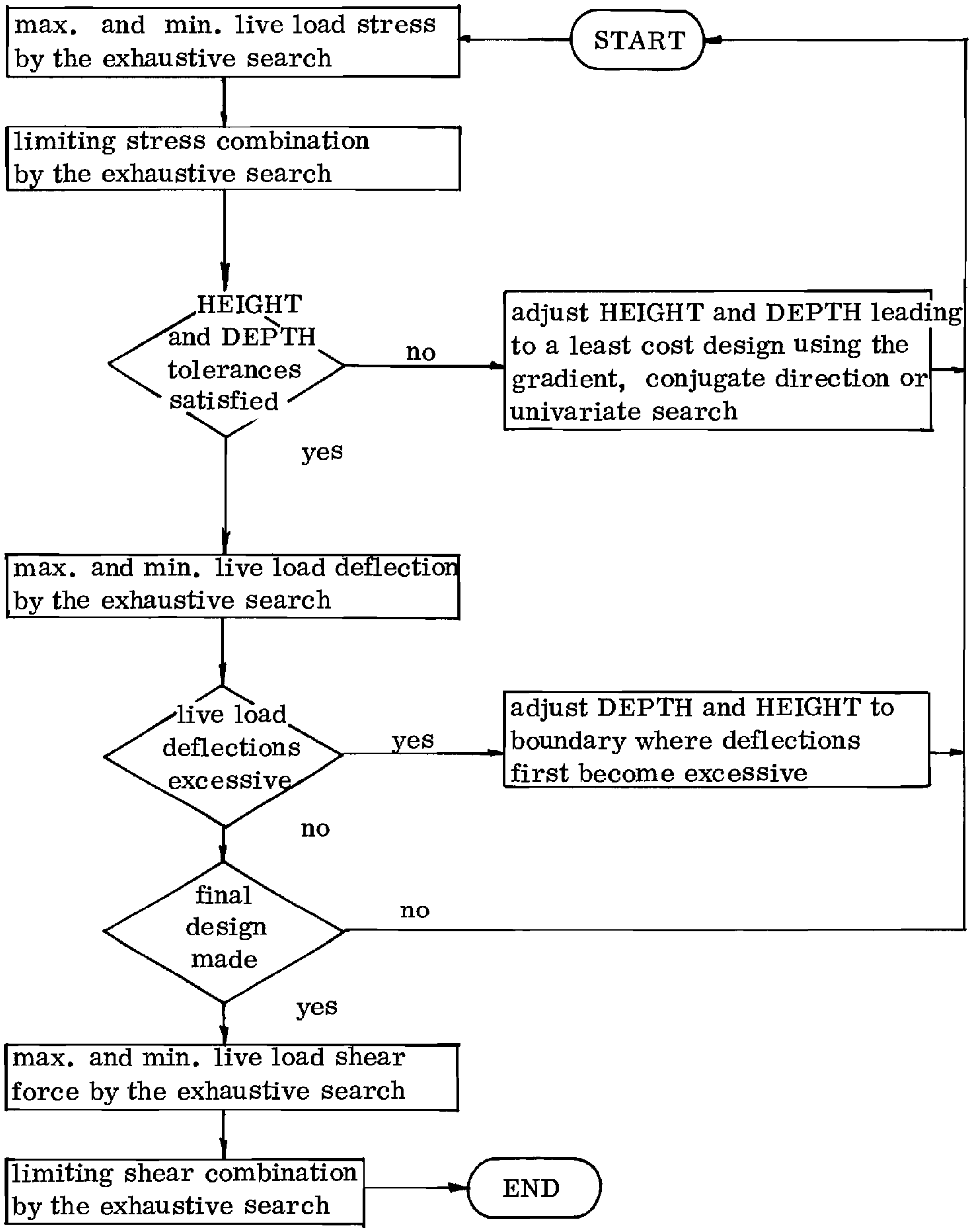


Fig. 6. Search Plan Using the Multivariate Searches

with a Stepping Procedure.

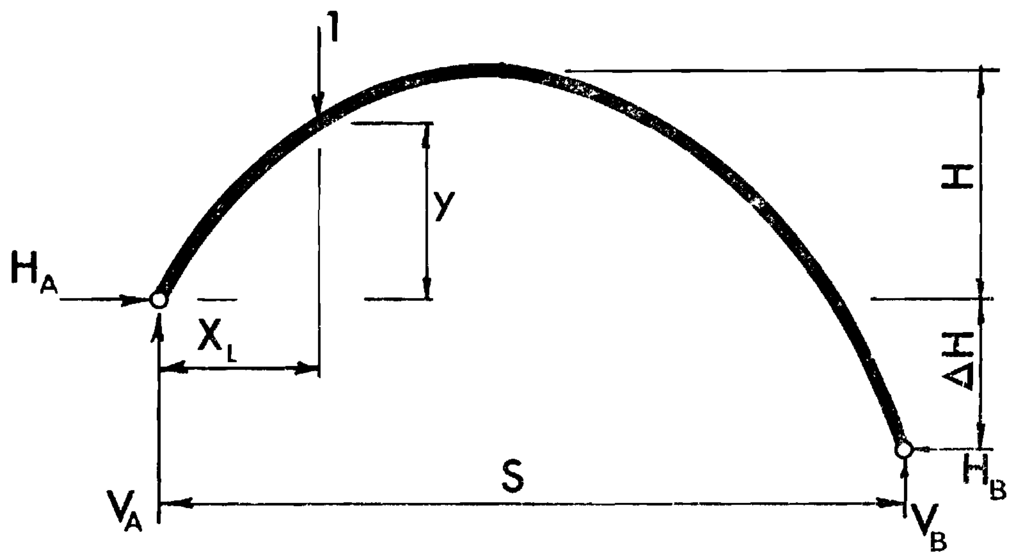


Fig. 7. Arch With Unit Load.

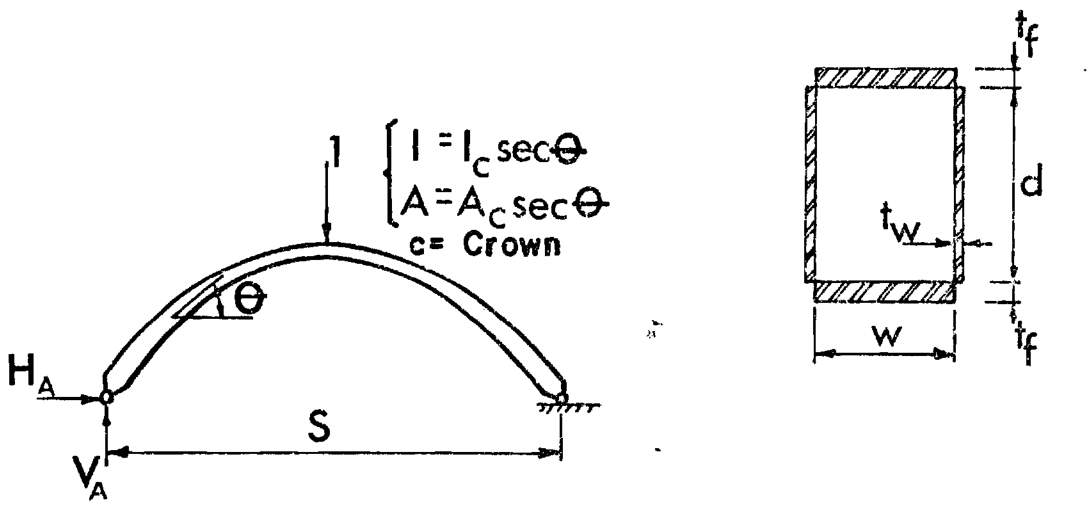


Fig. 8. Symmetrical Parabolic Arch with a Secant Varying Moment of Inertia and Area.

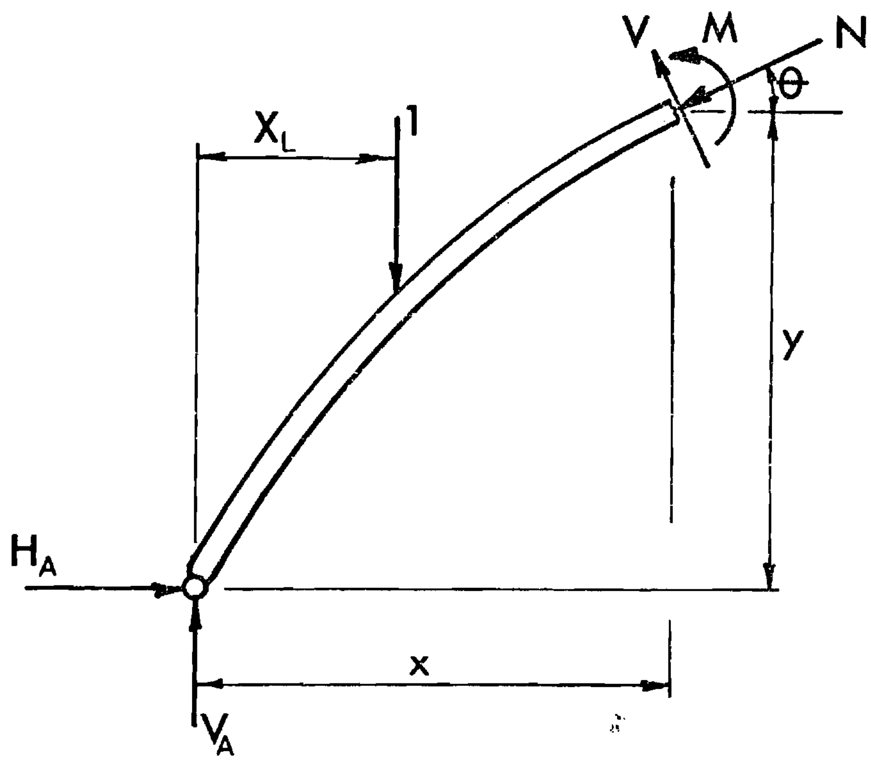
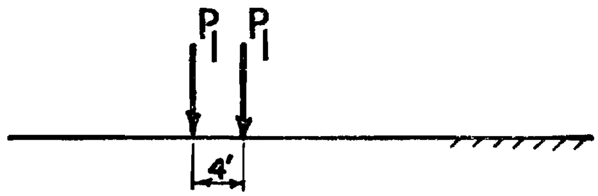
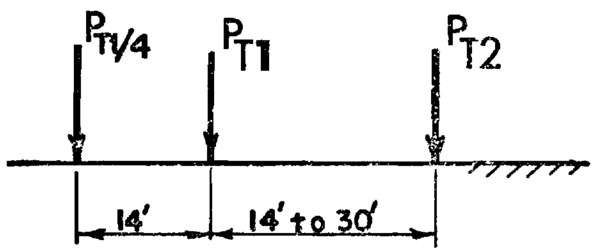


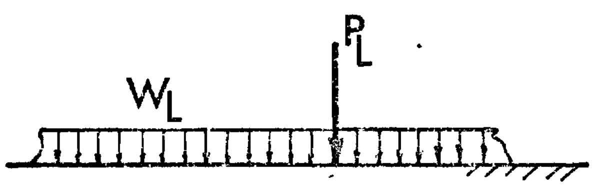
Fig. 9. Free Body Diagram for Finding Influence Lines for M, N and V at Design Points.



a. Interstate Live Load

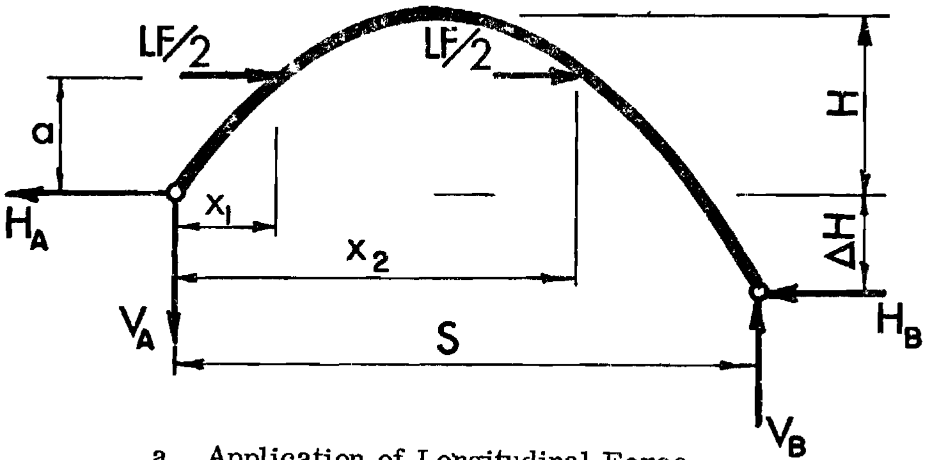


b. Truck Live Load

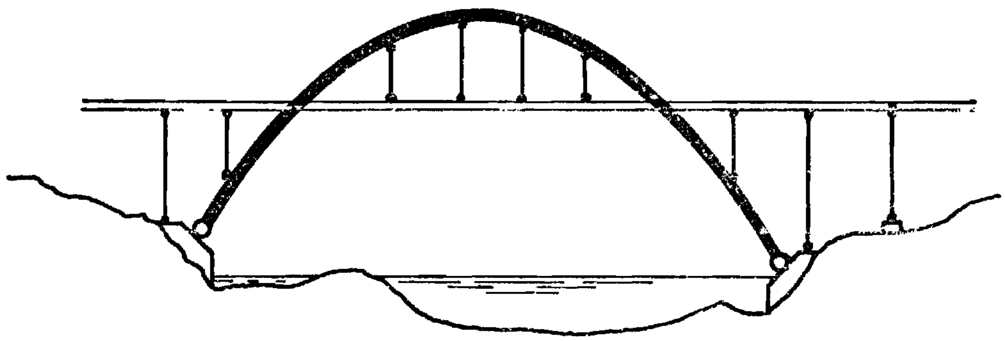


c. Lane Live Load

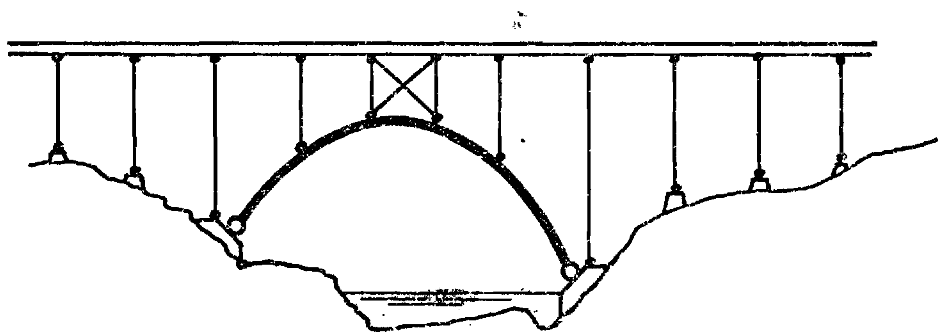
Fig. 10. Live Loads.



a. Application of Longitudinal Force



b. LF can be transferred to the Arch Rib where Deck and Arch Rib Cross



c. LF can be transferred to the Arch Rib by Column Bracing

Fig. 11. Longitudinal Force.

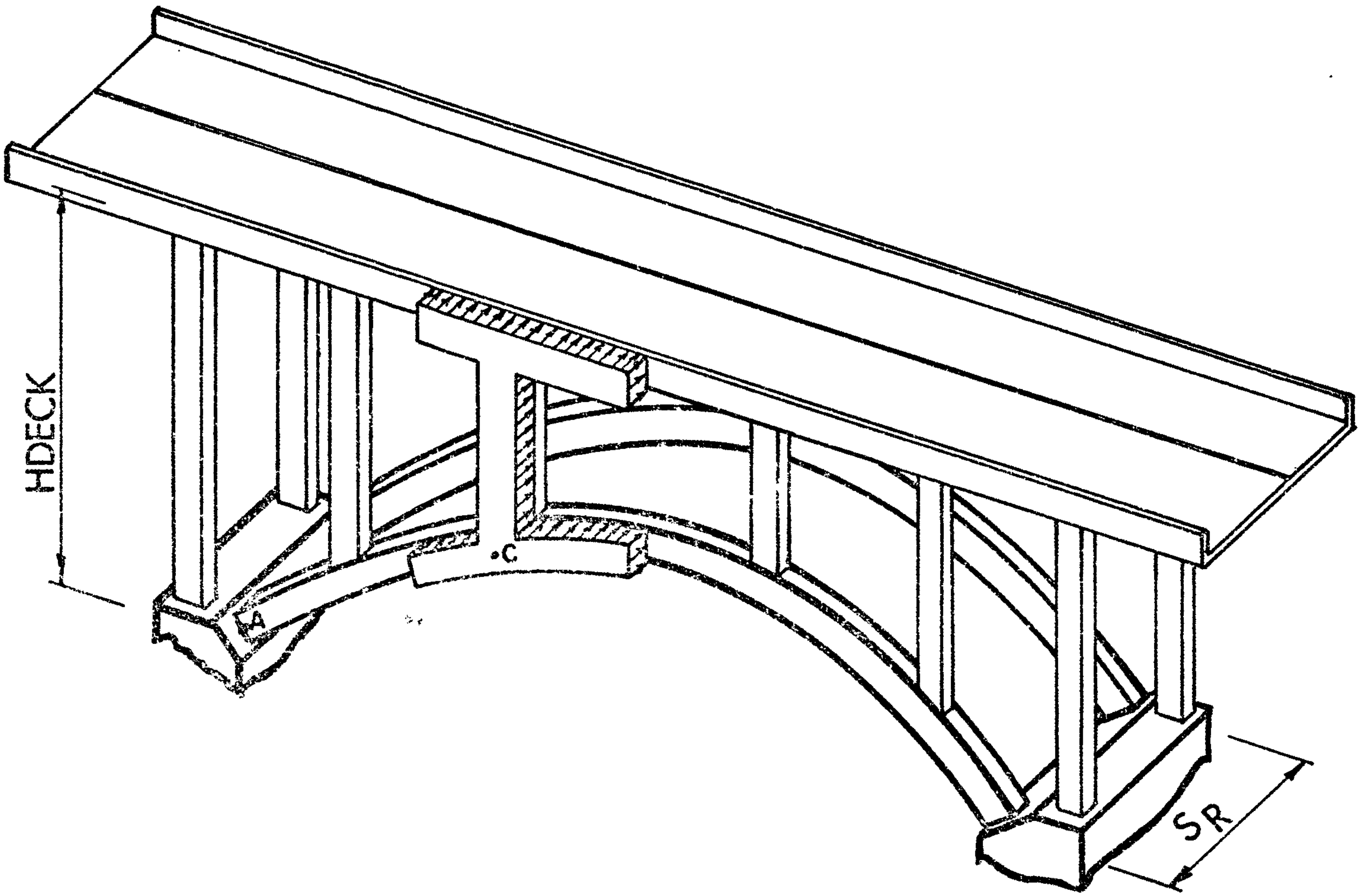


Figure 12. Wind Load at One Column.

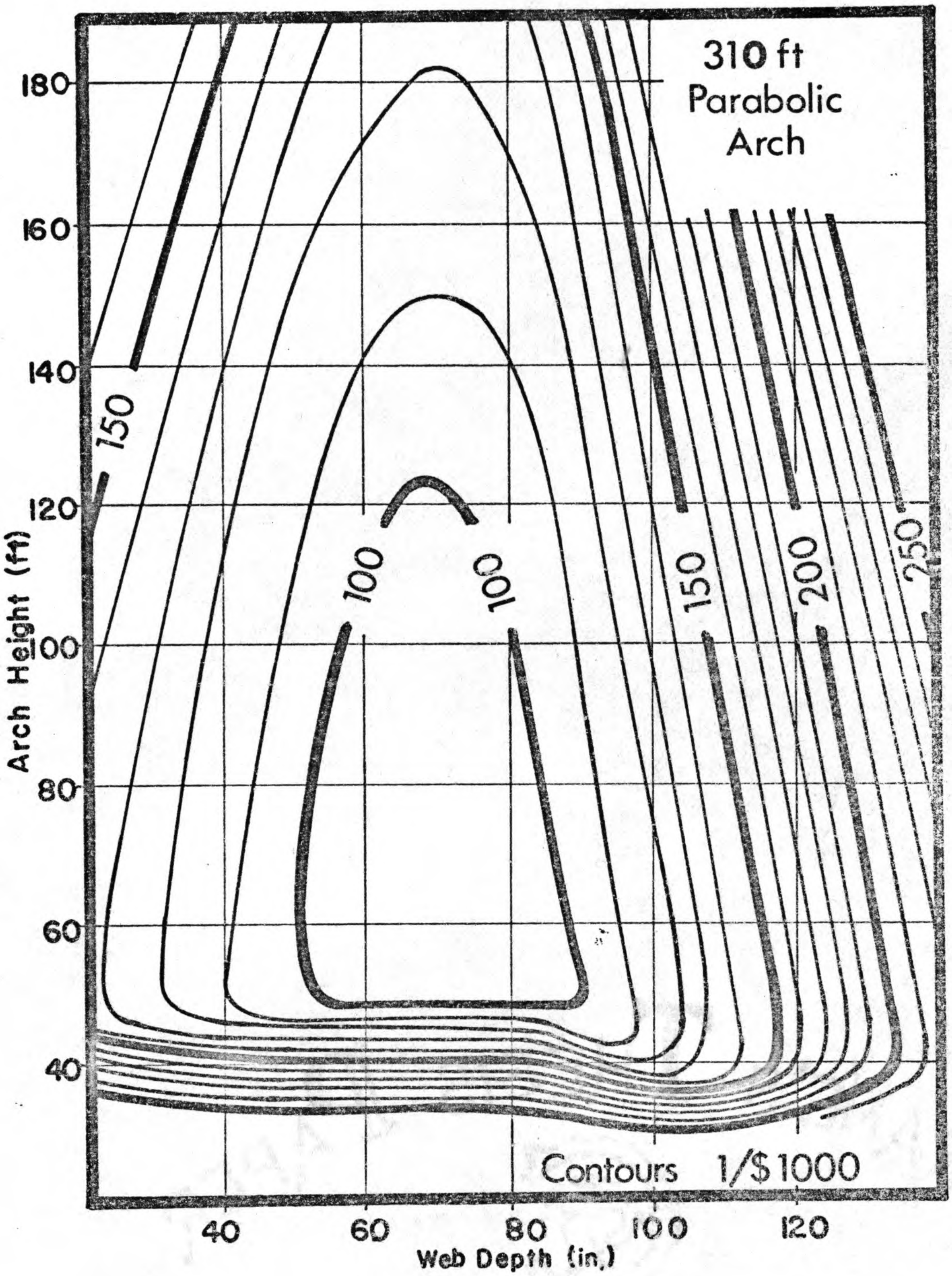


Fig. 13. Cost Contours for Objective Surface.

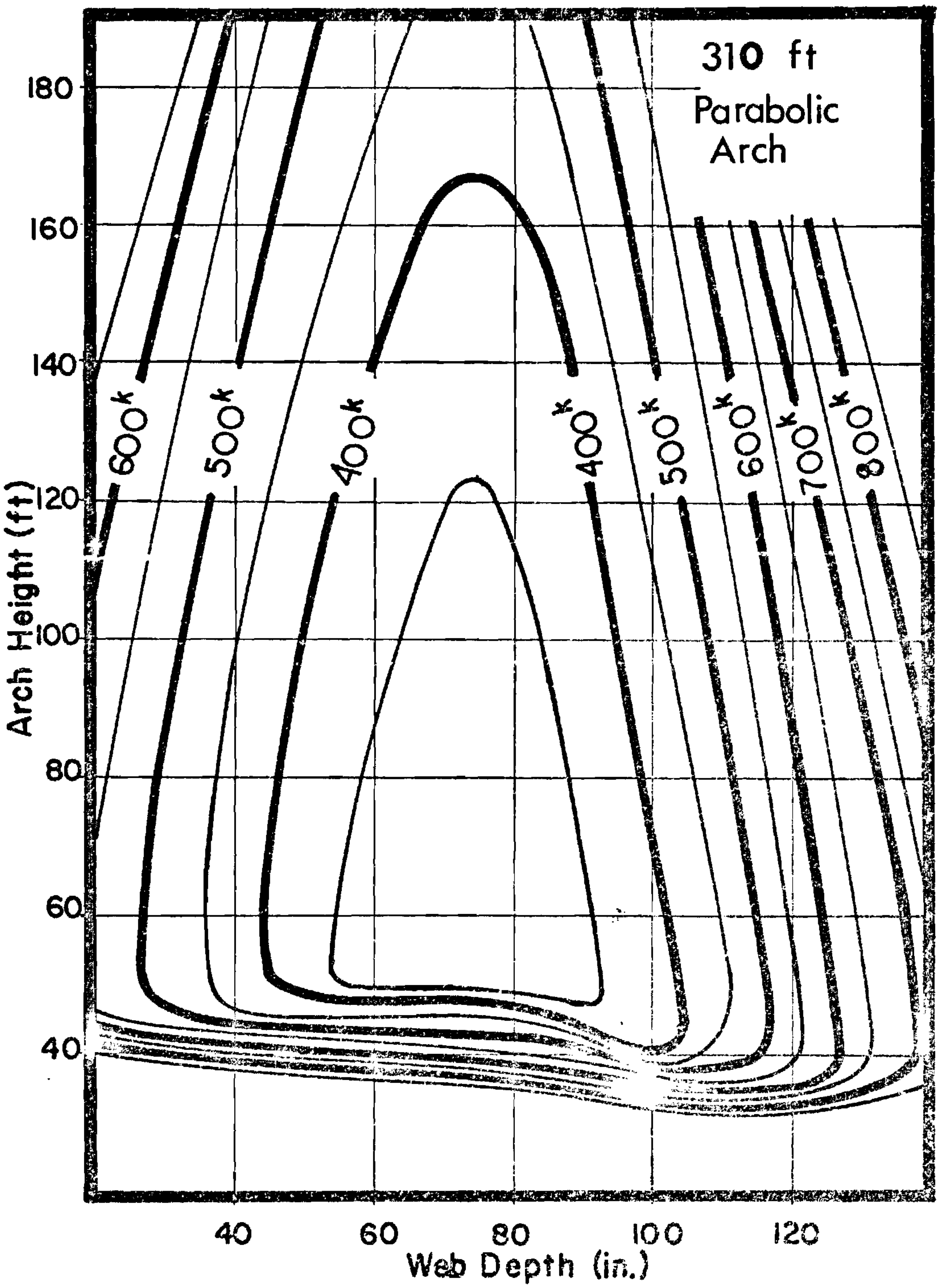


Fig. 14. Flange and Web Weight Contours.

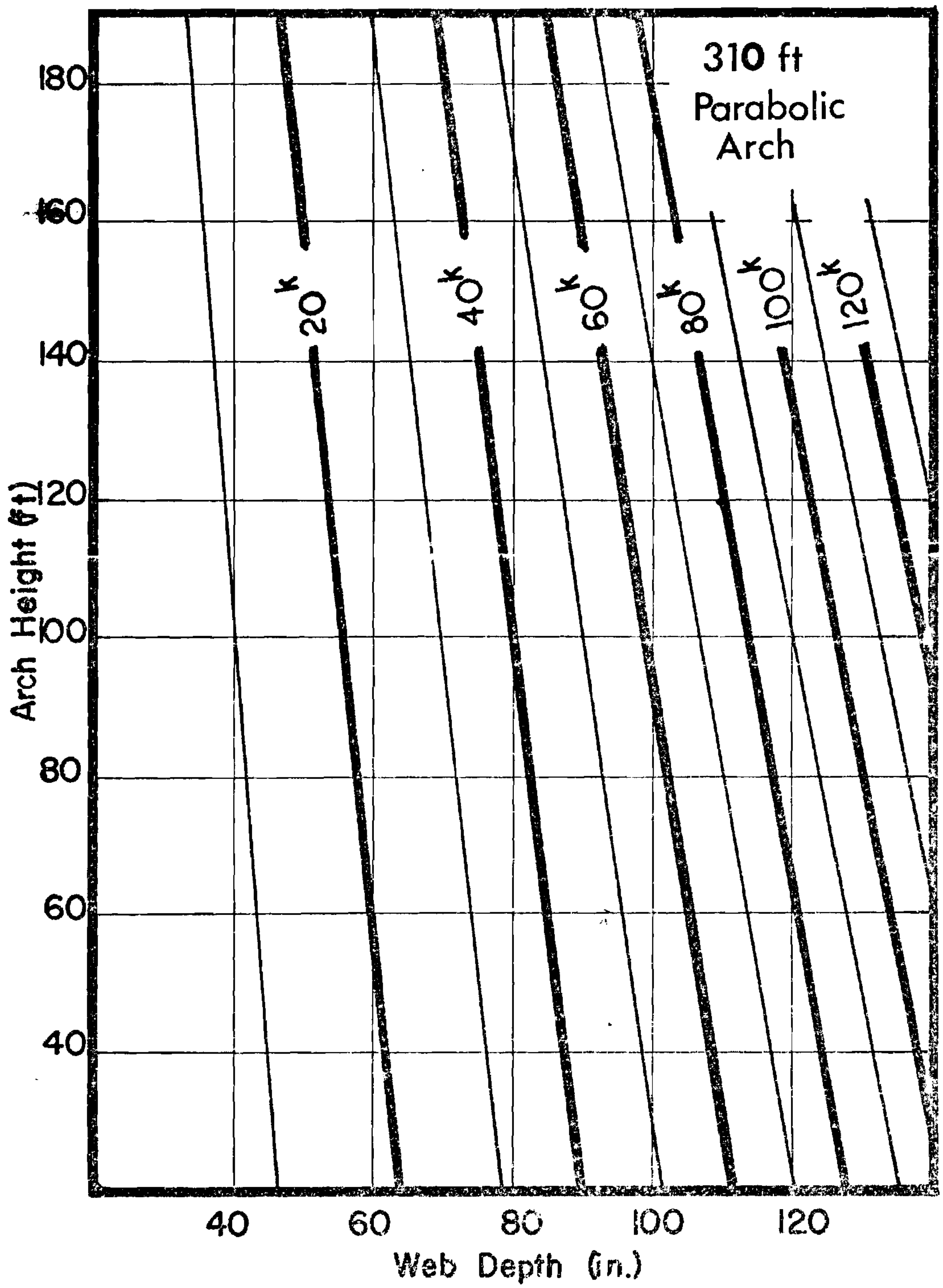


Fig. 15. Diaphragm and Stiffener Weight Contours.

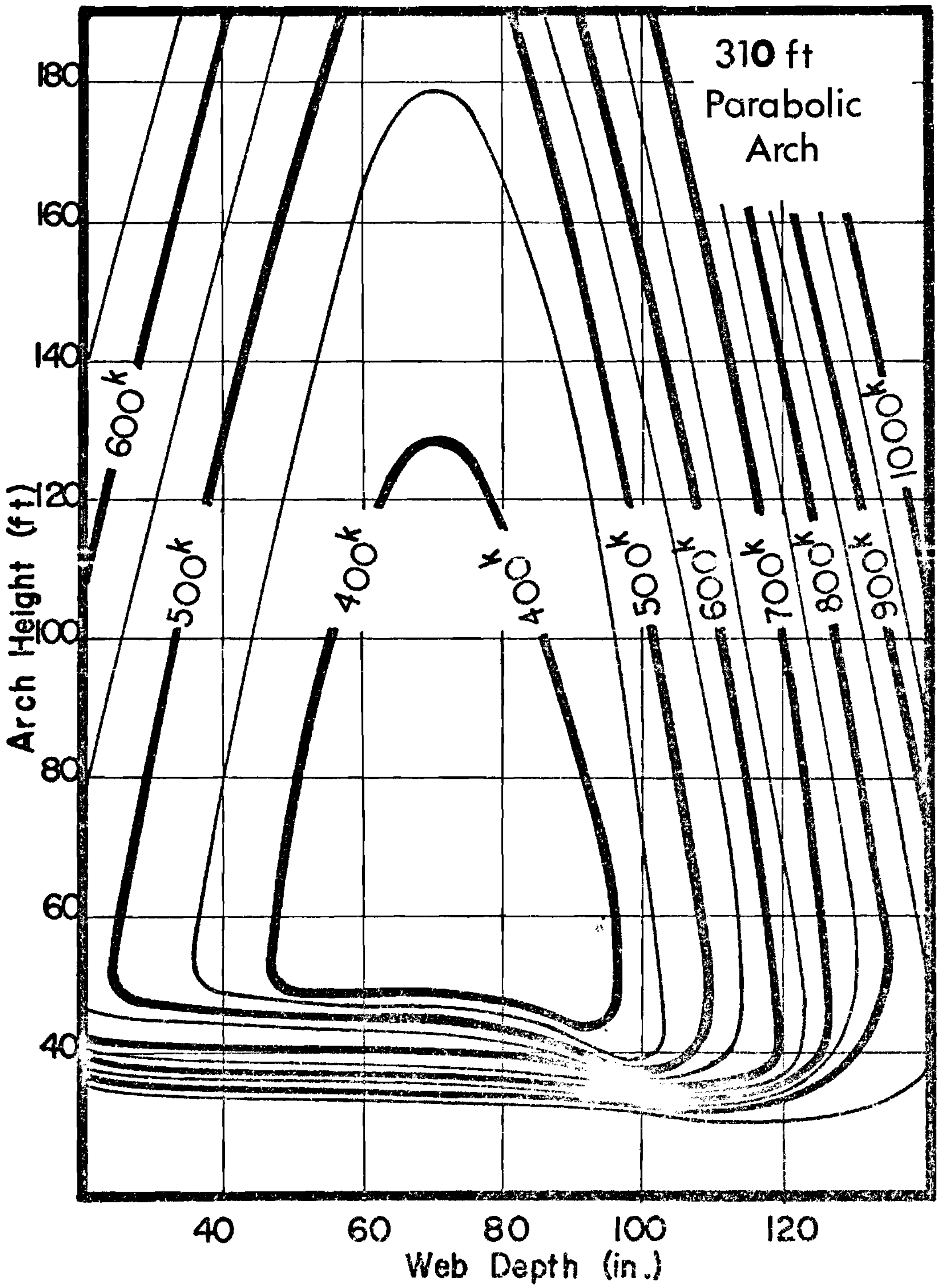


Fig. 16. Total Weight Contours.

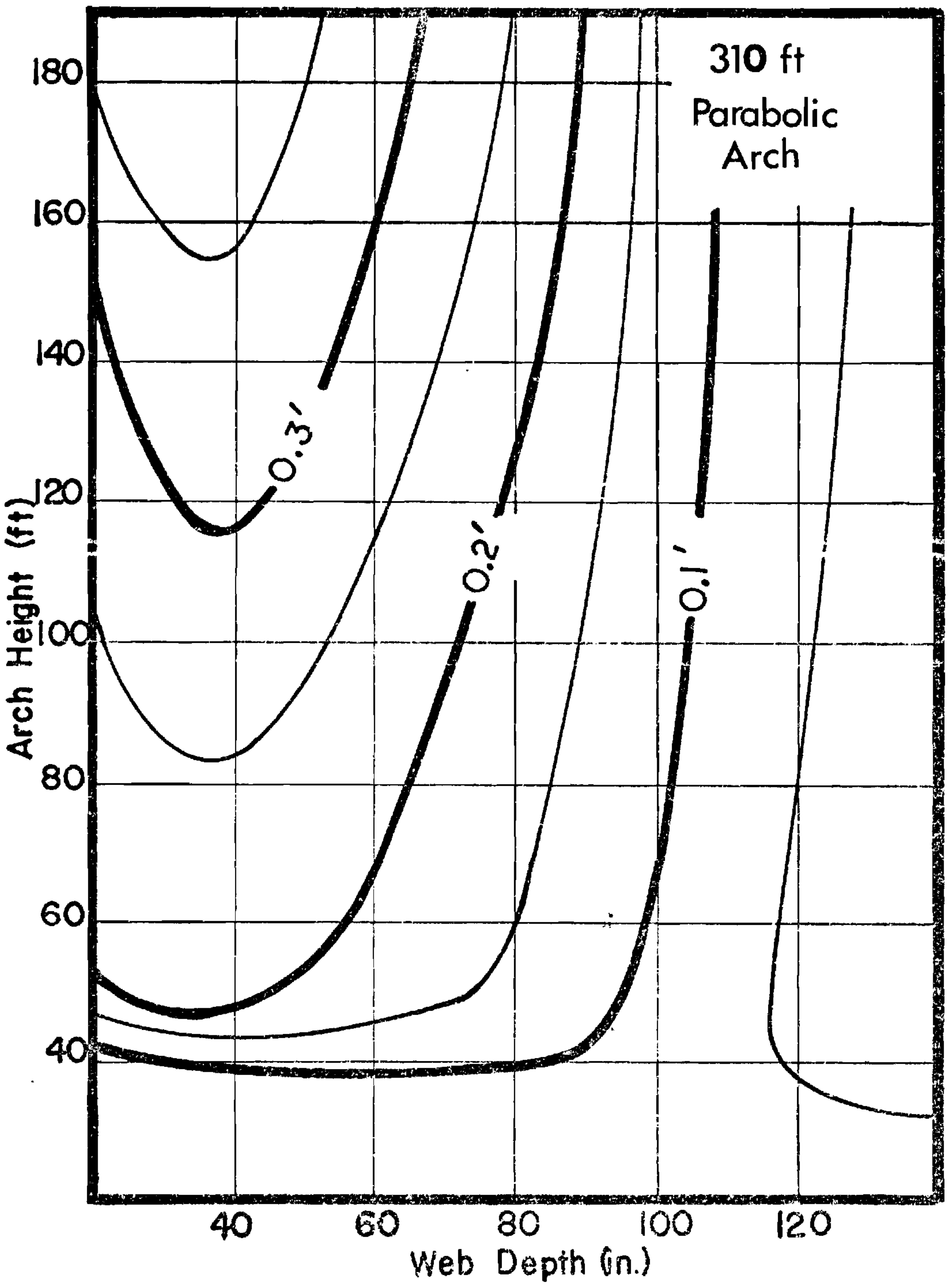


Fig. 17. Maximum Downward Live Load Deflection Contours.

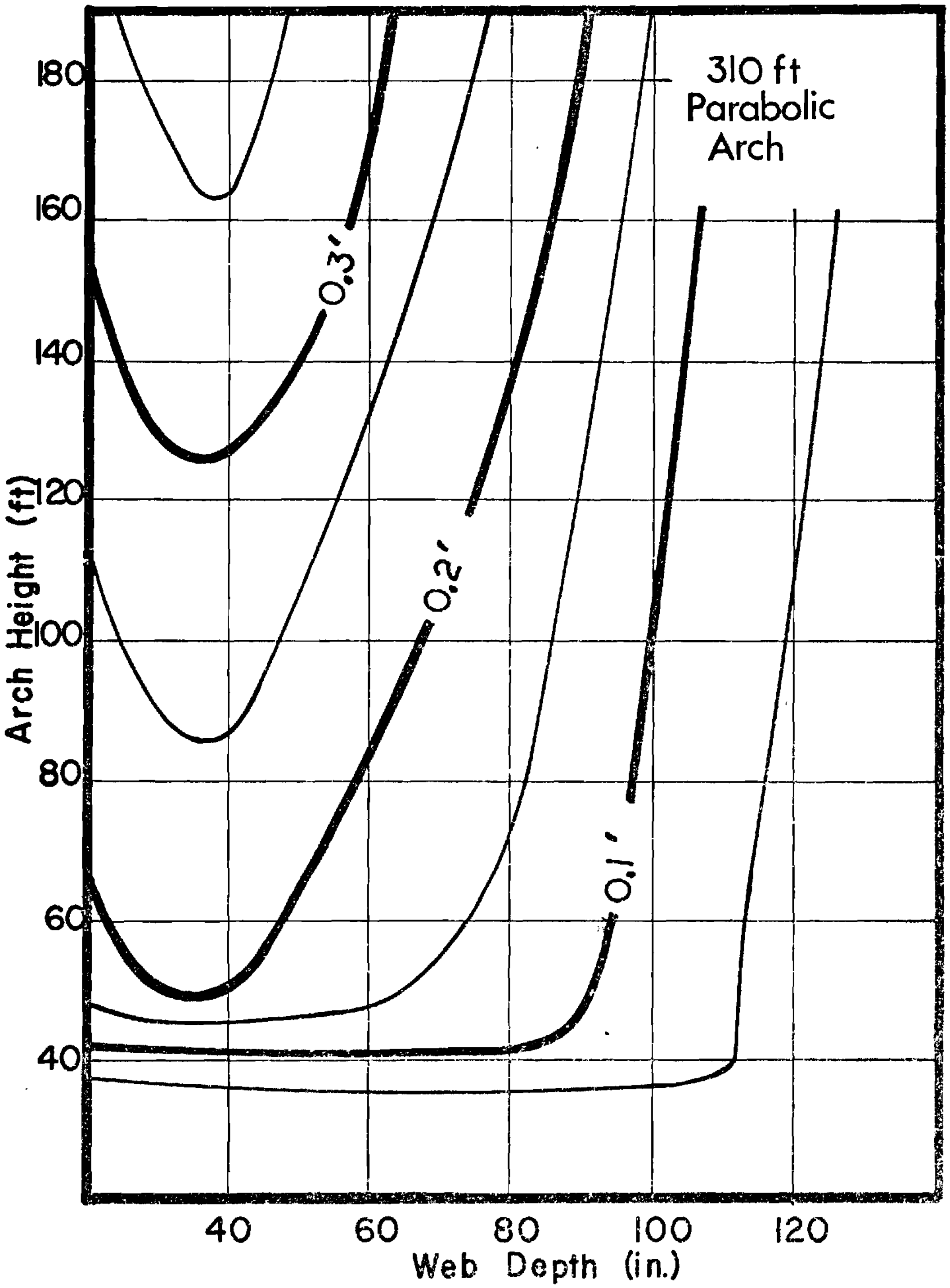


Fig. 18. Maximum Upward Live Load Deflection Contours.

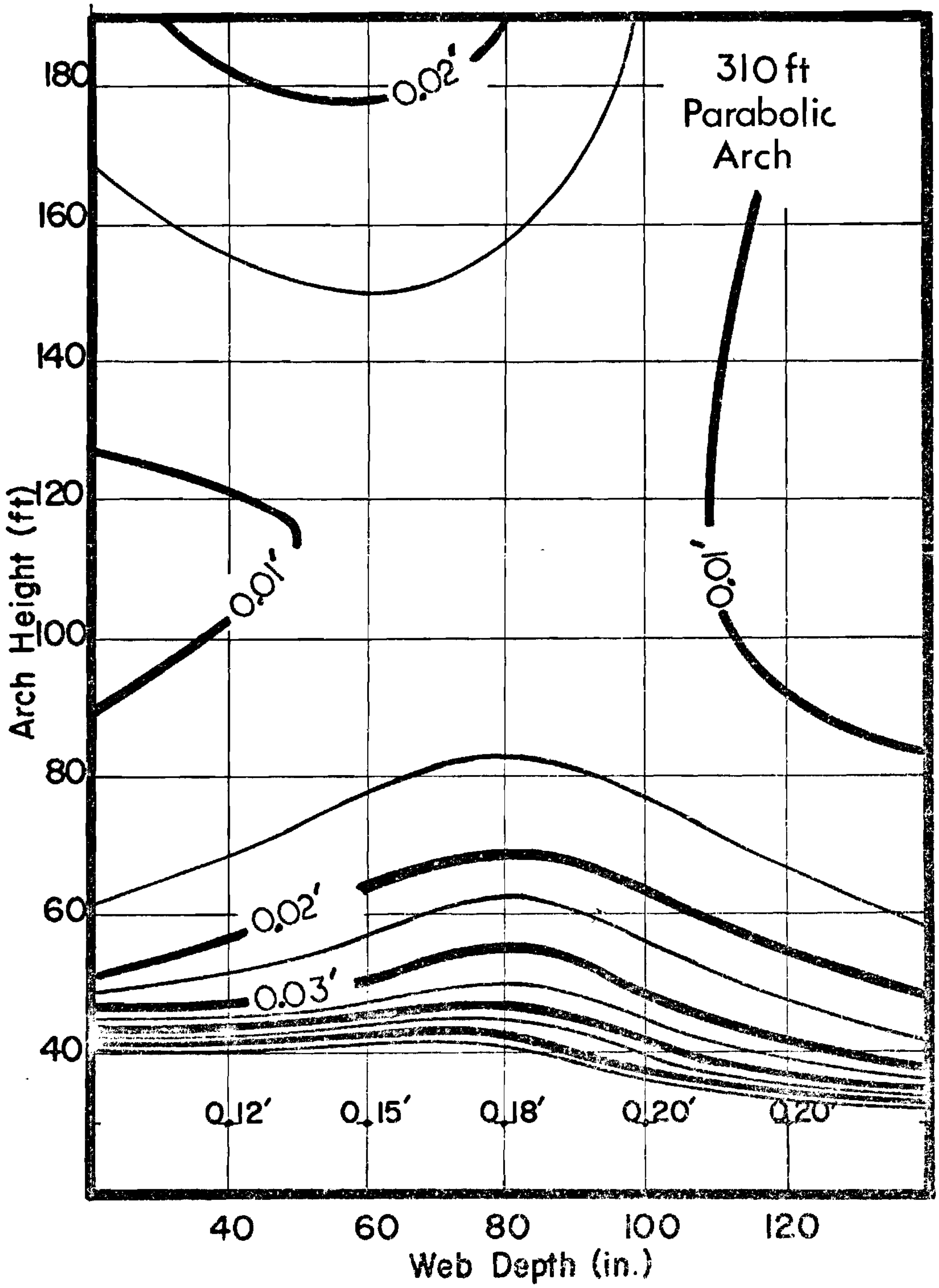


Fig. 19. Maximum Dead Load Deflection Contours.

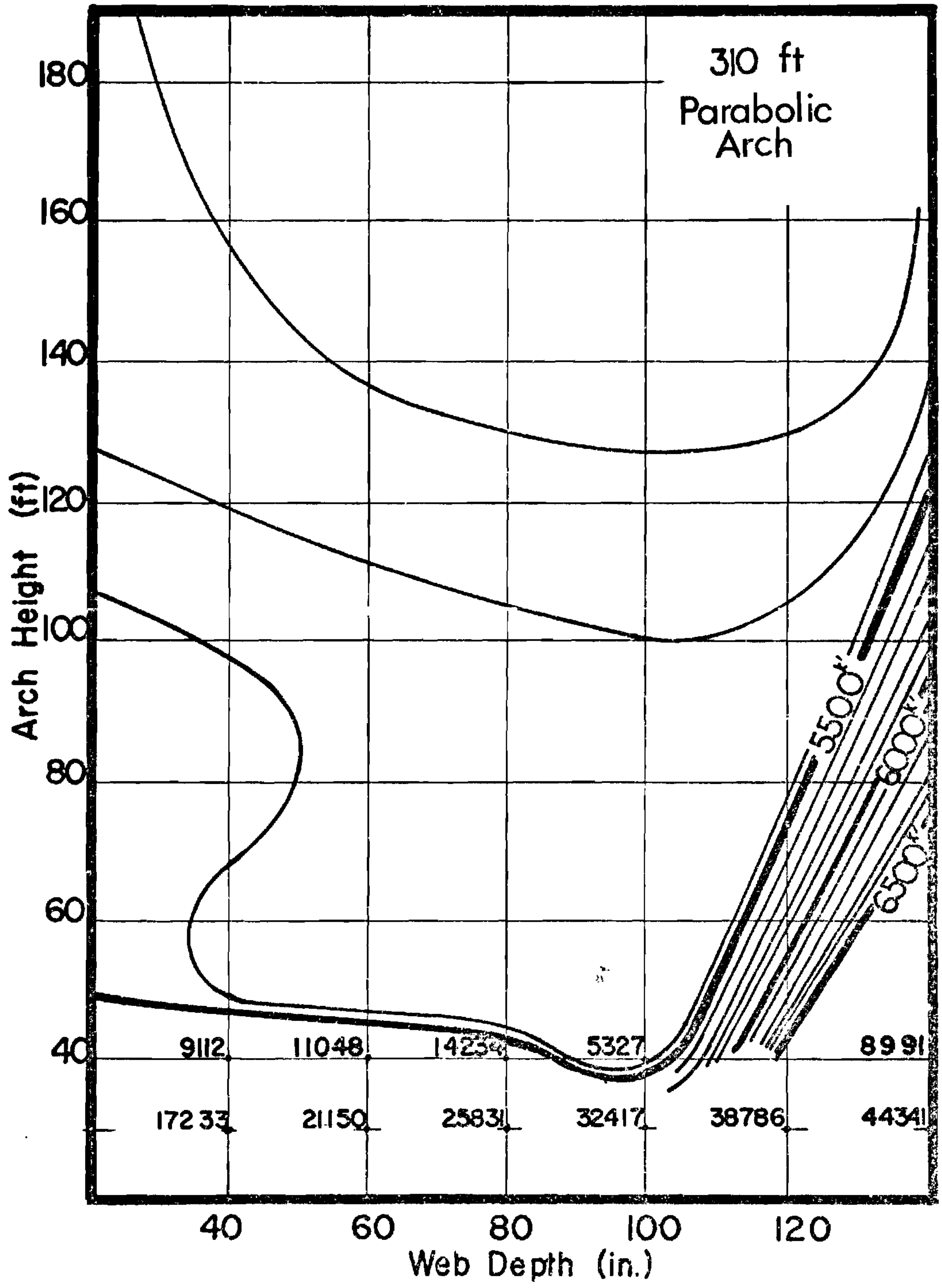


Fig. 20. Maximum Moment Contours.

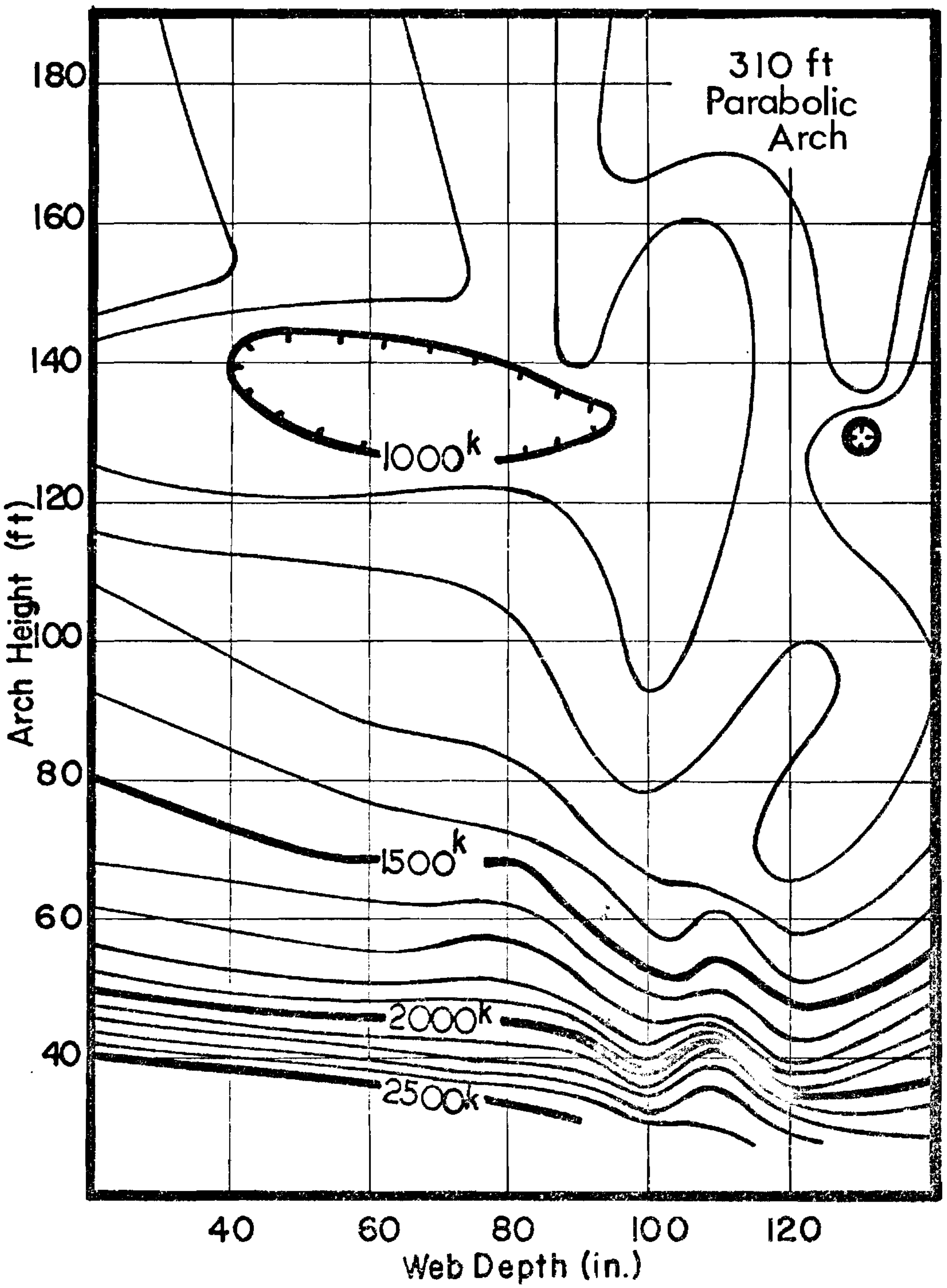


Fig. 21. Maximum Thrust Contours.

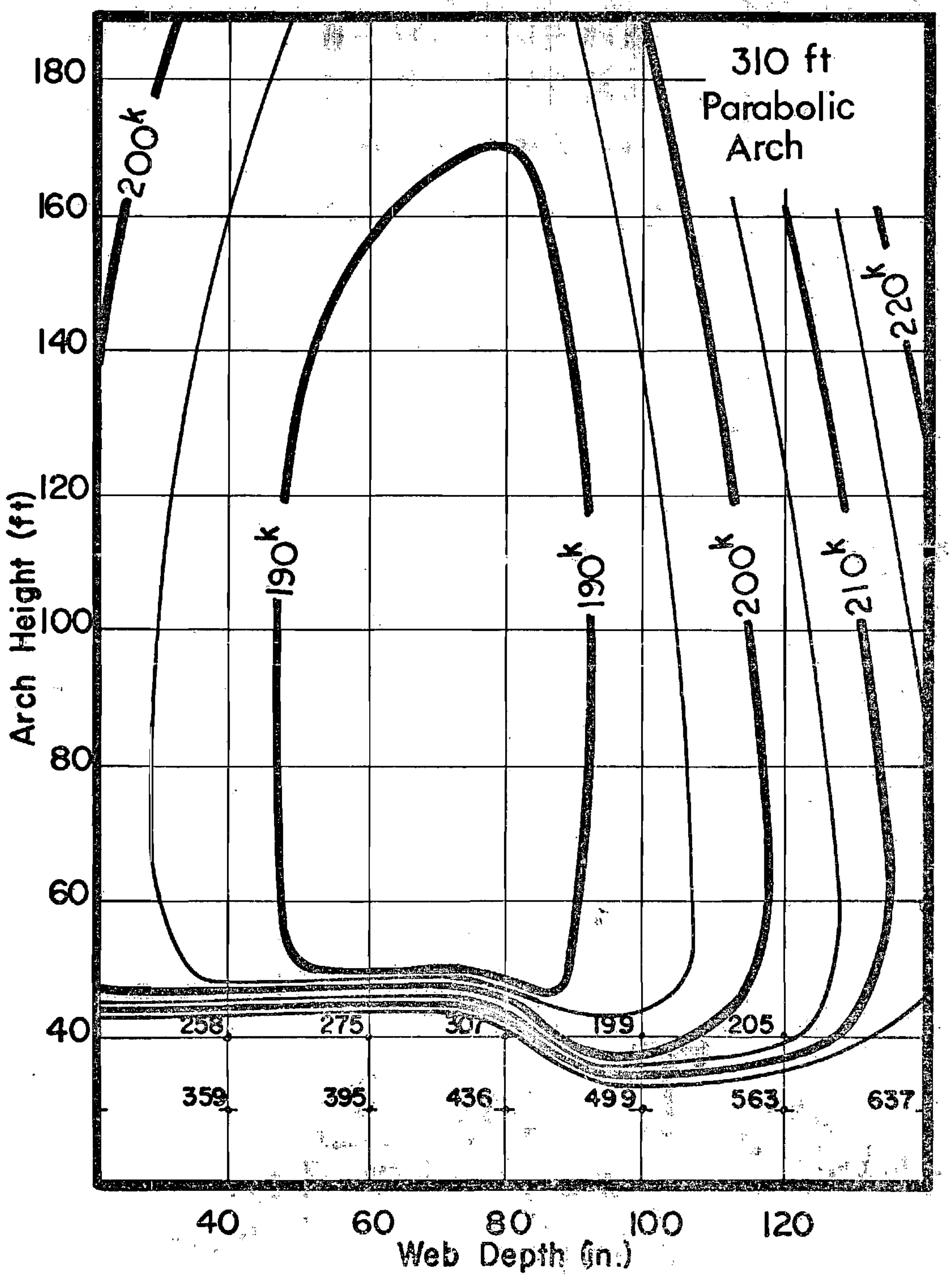


Fig. 22. Maximum Shear Contours.

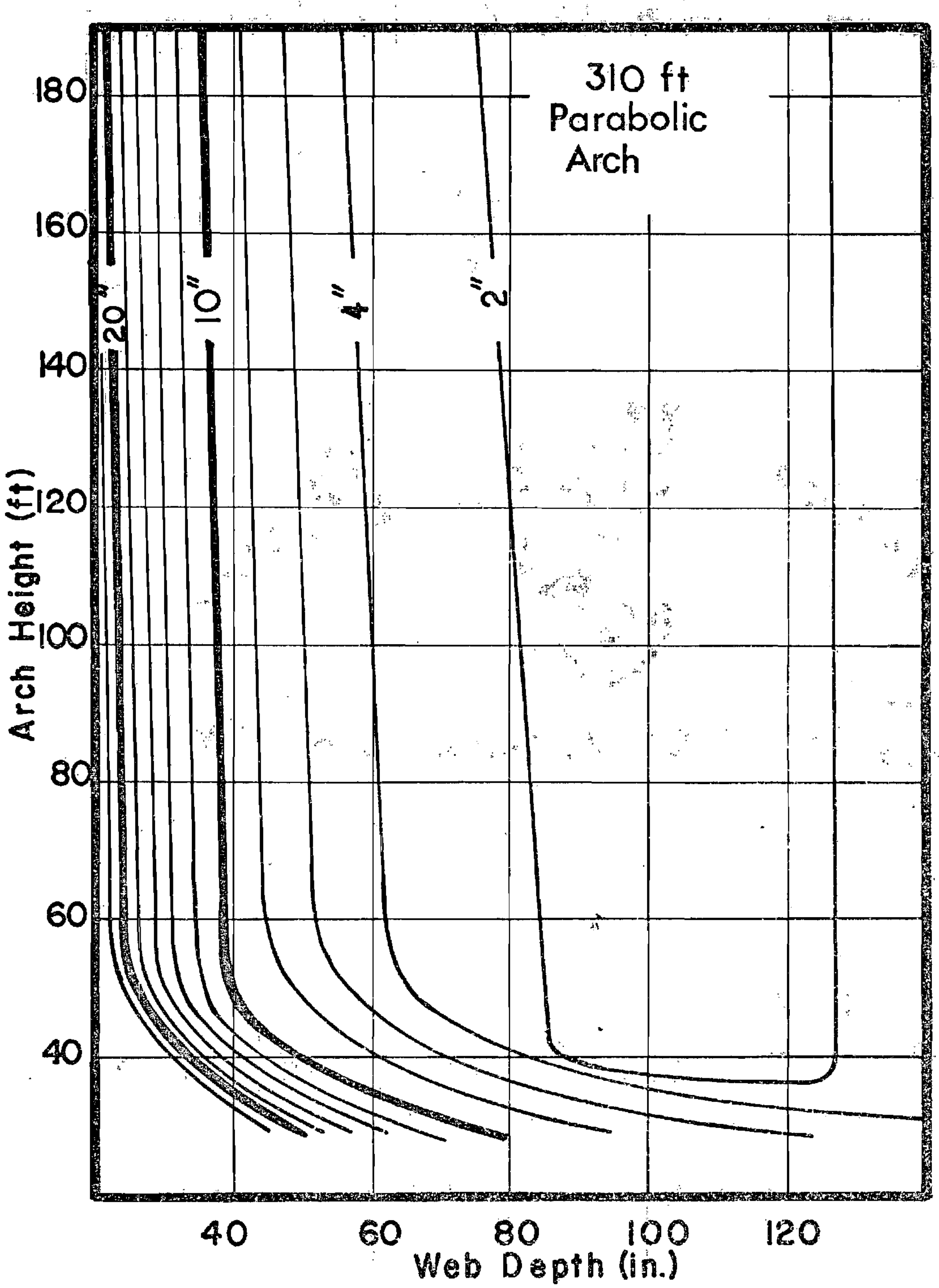


Fig. 23. Maximum Flange Thickness Contours.

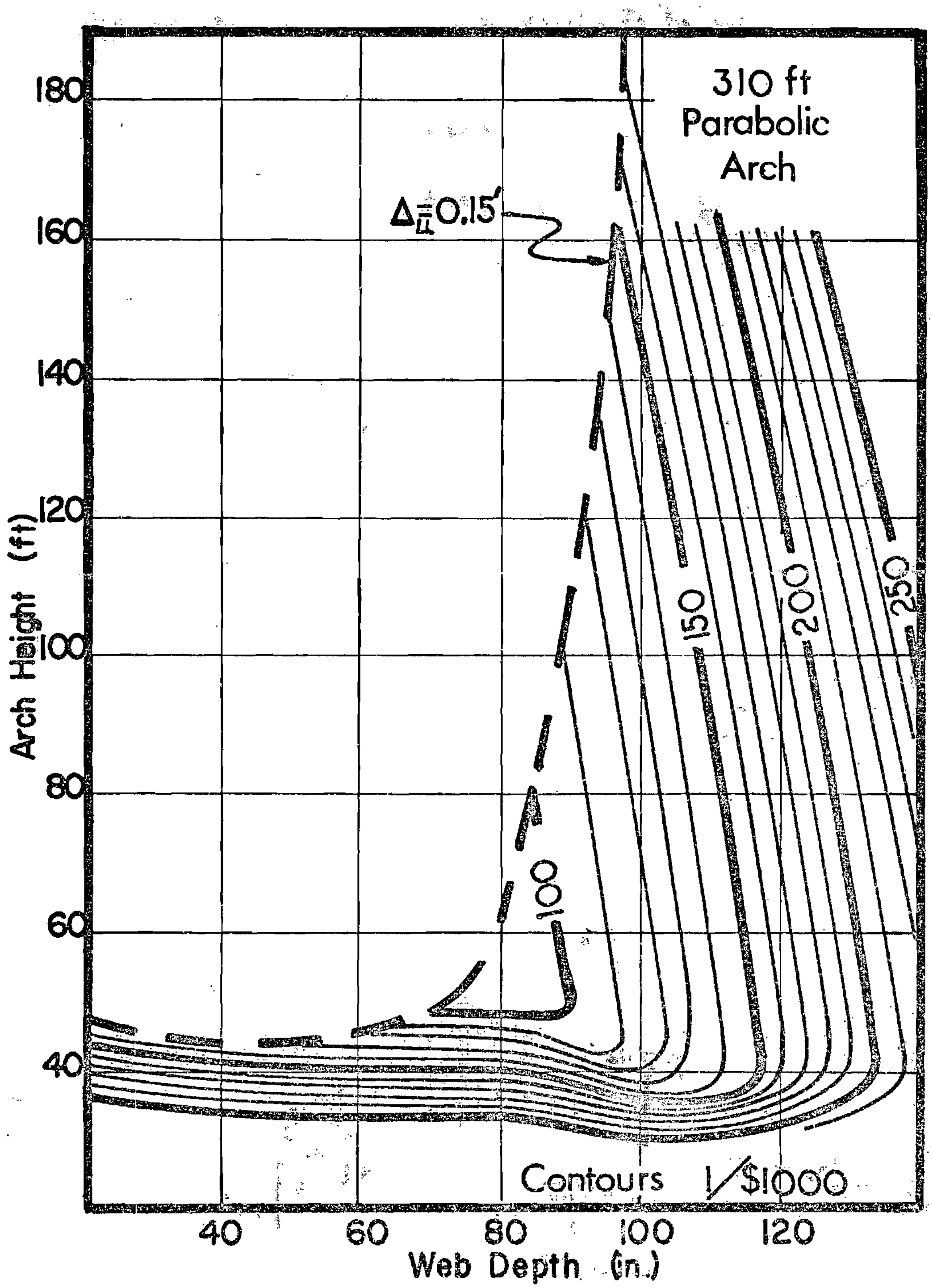


Fig. 24. Restricted Cost Objective for a Possible Live Load Deflection Constraint ($\Delta \leq 0.15$ ft).

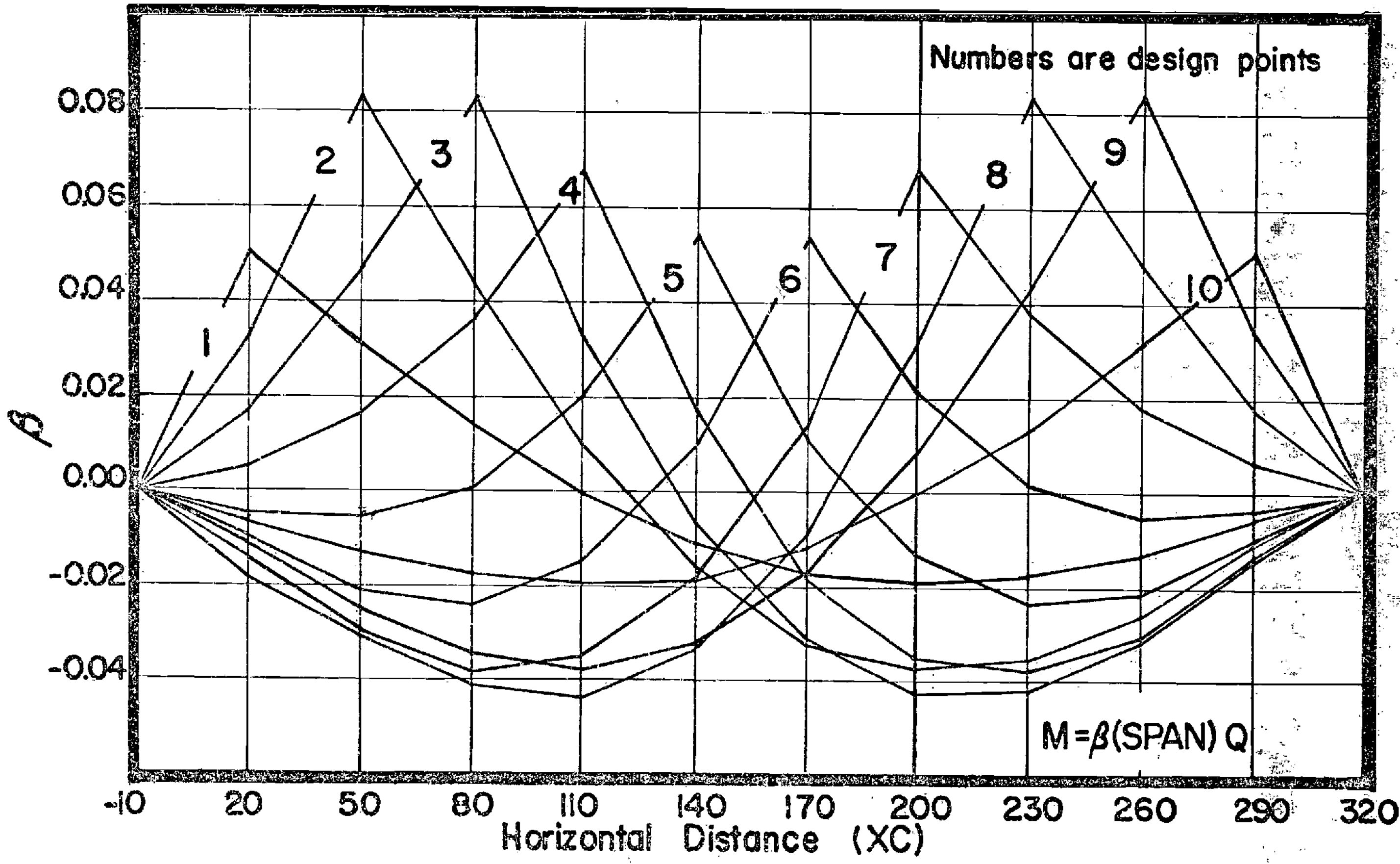


Fig. 25. Moment Influence Lines for Arch of Table 3.

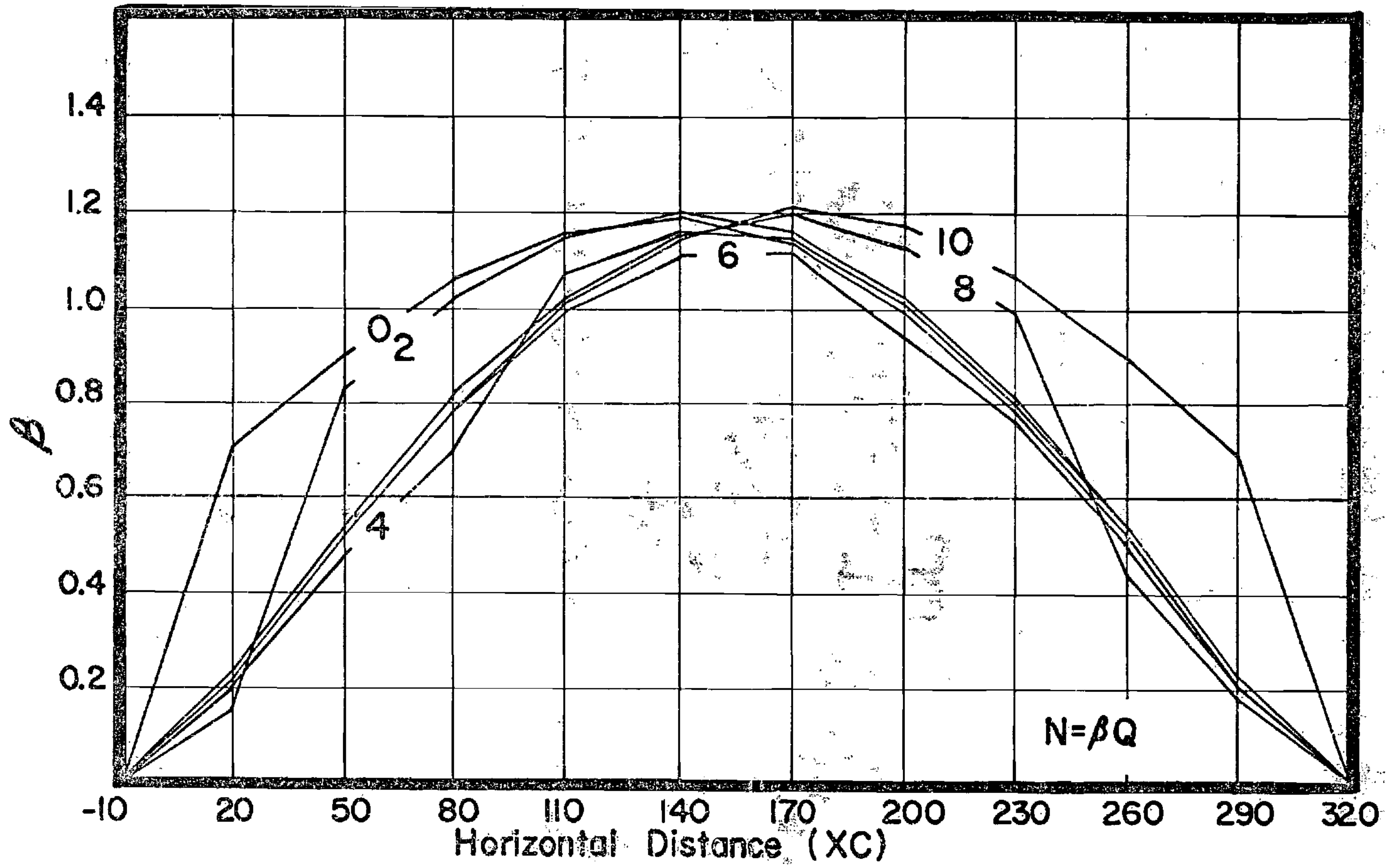


Fig. 26a. Thrust Influence Lines for Arch of Table 3.

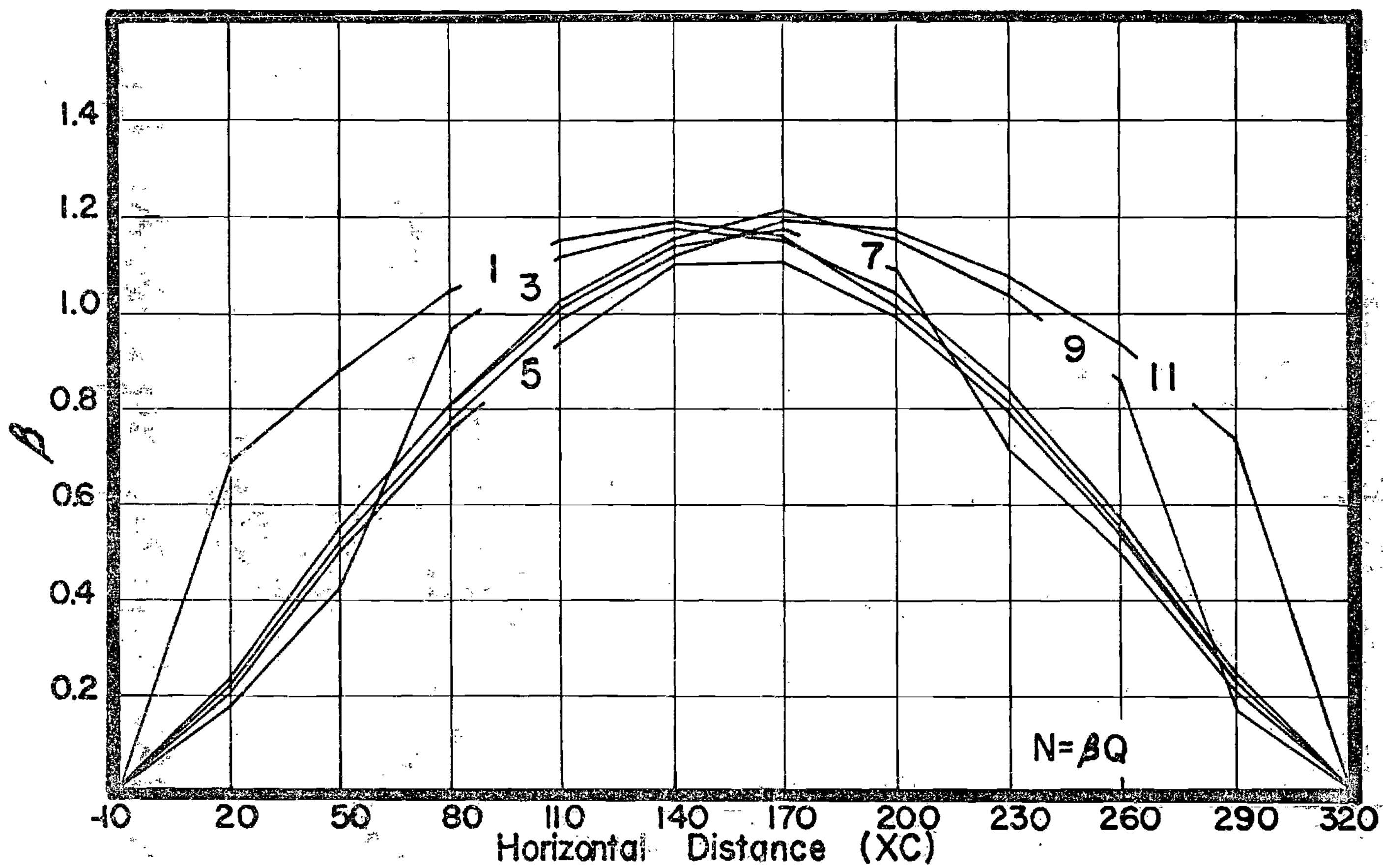


Fig. 26b. Thrust Influence Lines for Arch of Table 3.

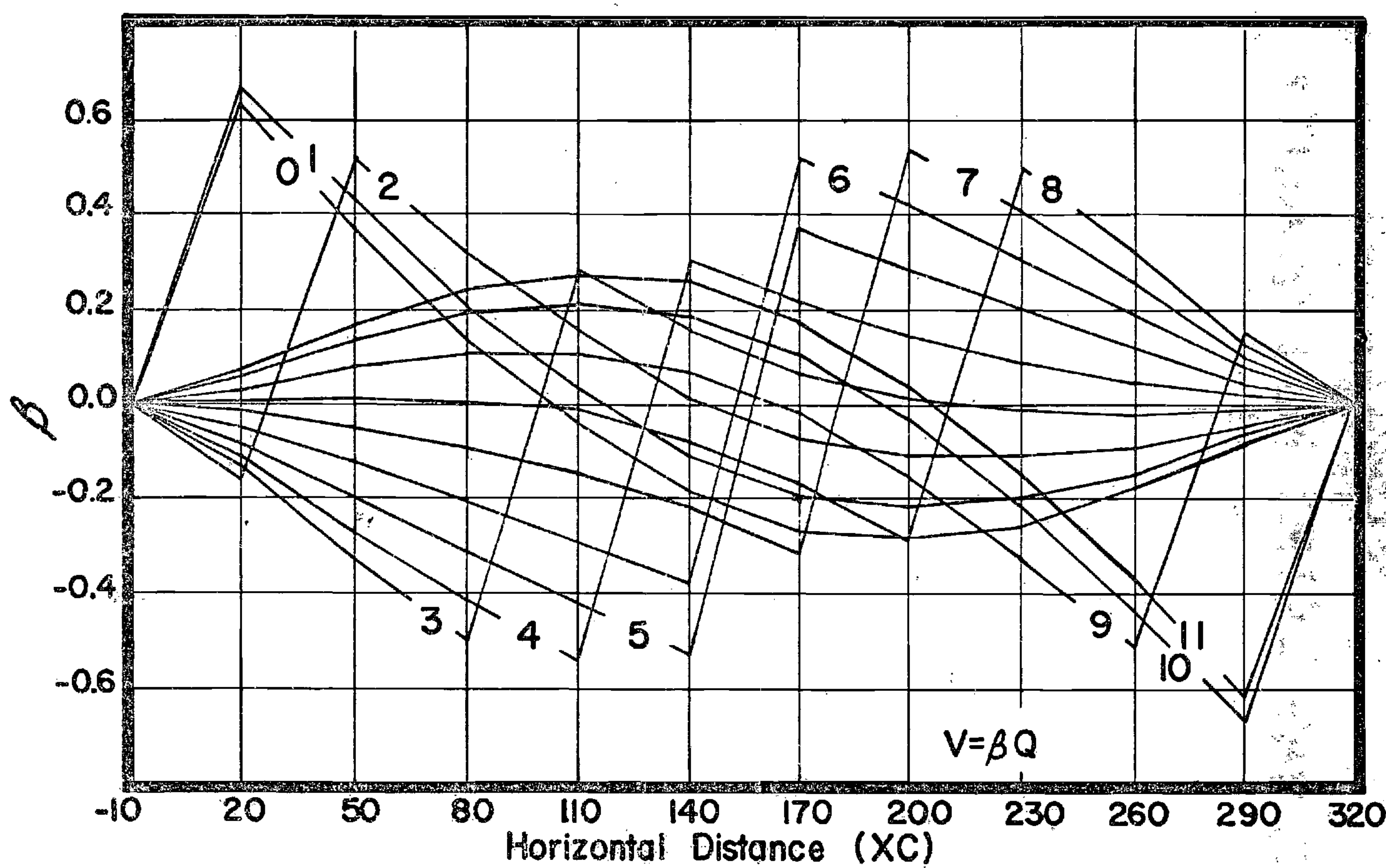


Fig. 27. Shear Influence Lines for Arch of Table 3.

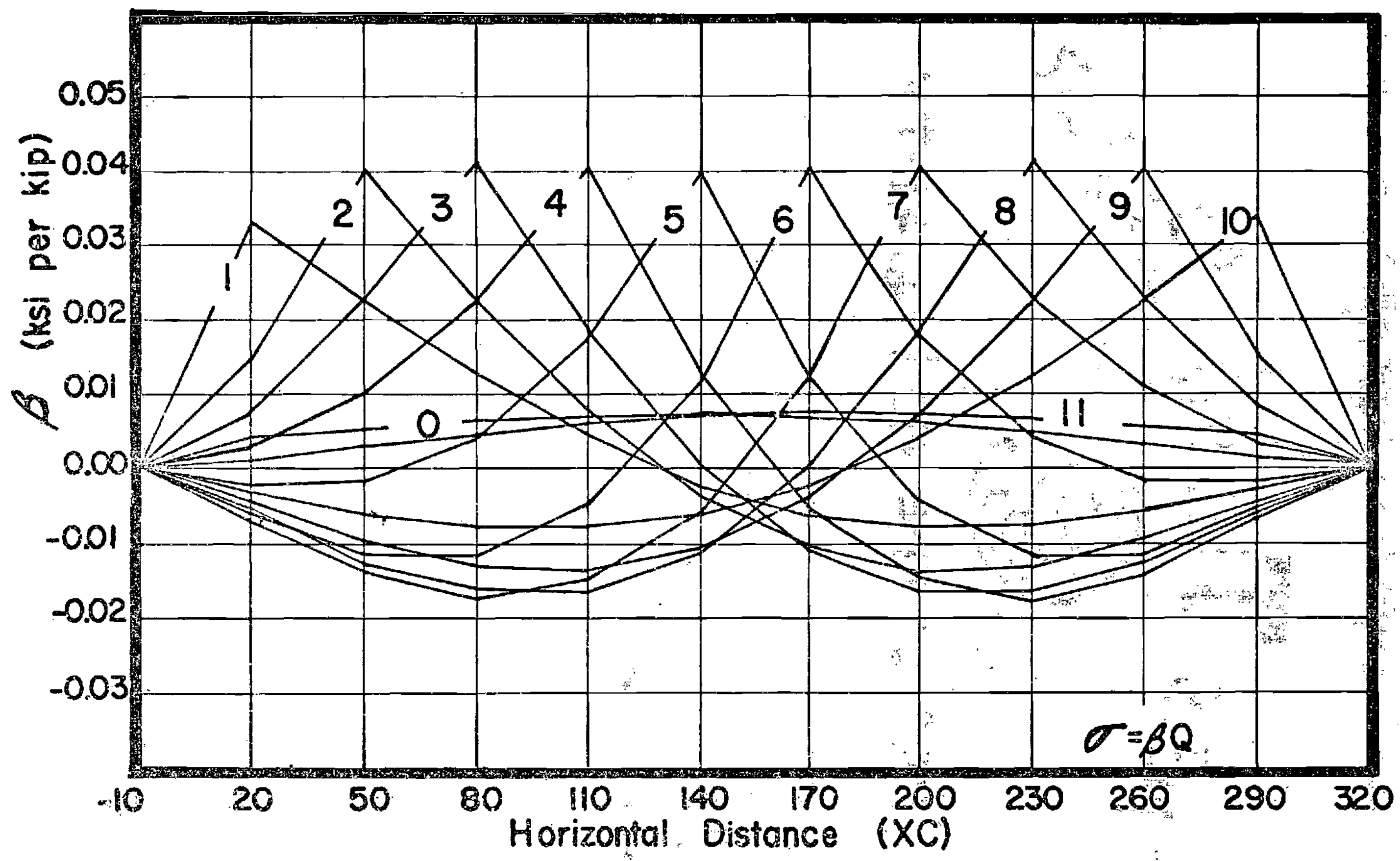


Fig. 28. Stress Influence Lines for Arch of Table 3.

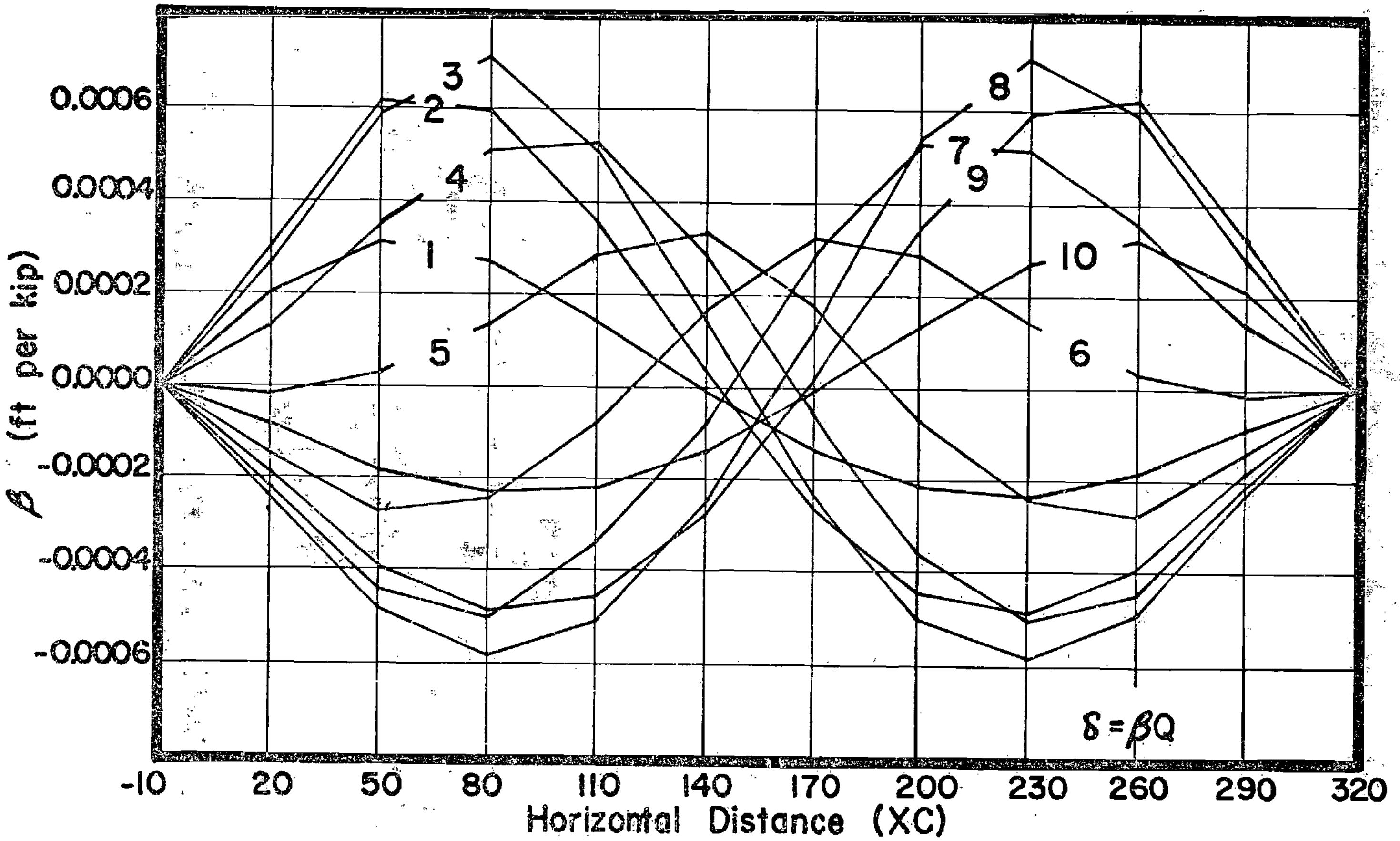


Fig. 29. Deflection Influence Lines for Arch of Table 3.

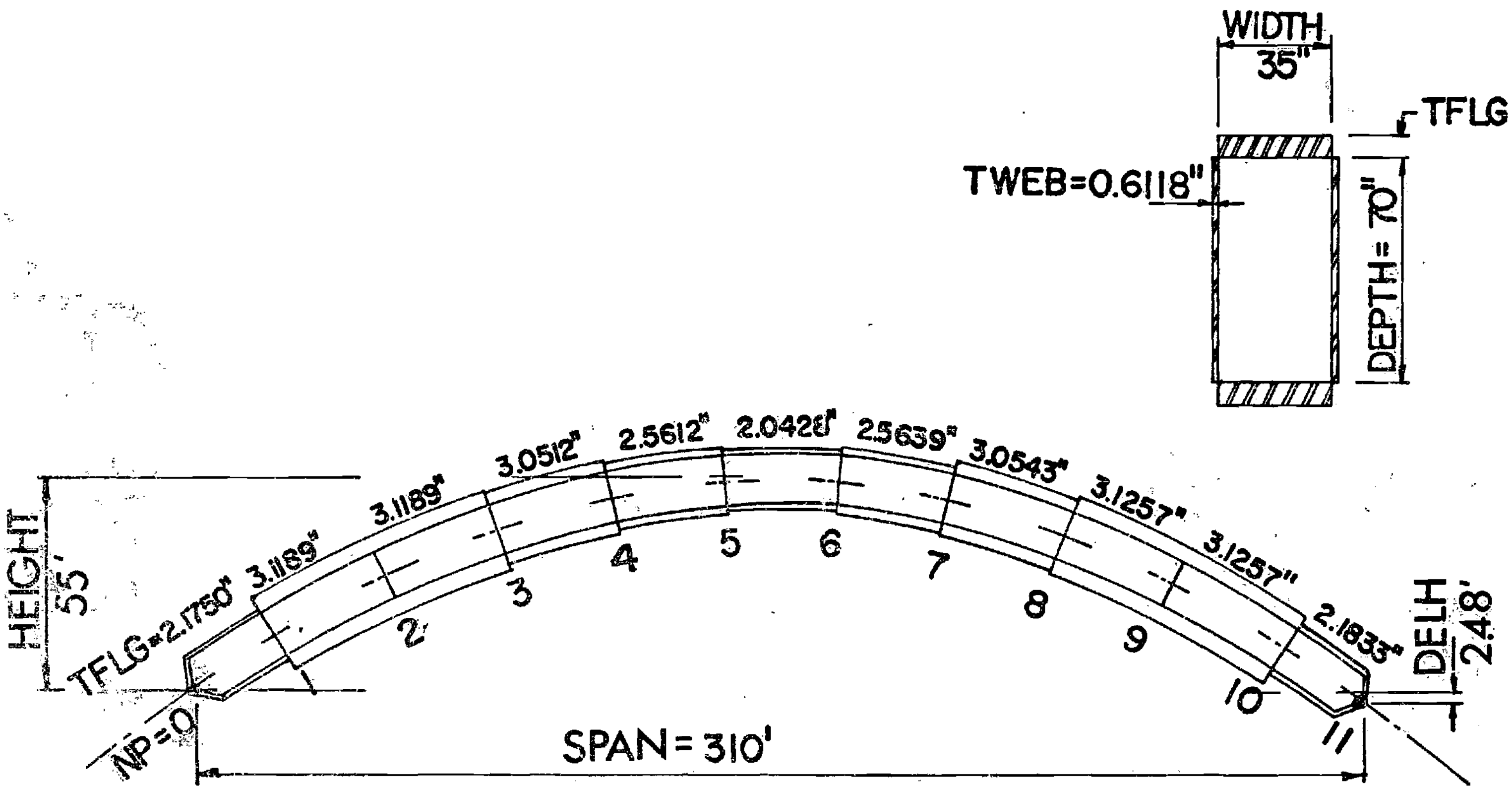


Fig. 30. Sketch of 310 ft Parabolic Arch Rib (Table 3).

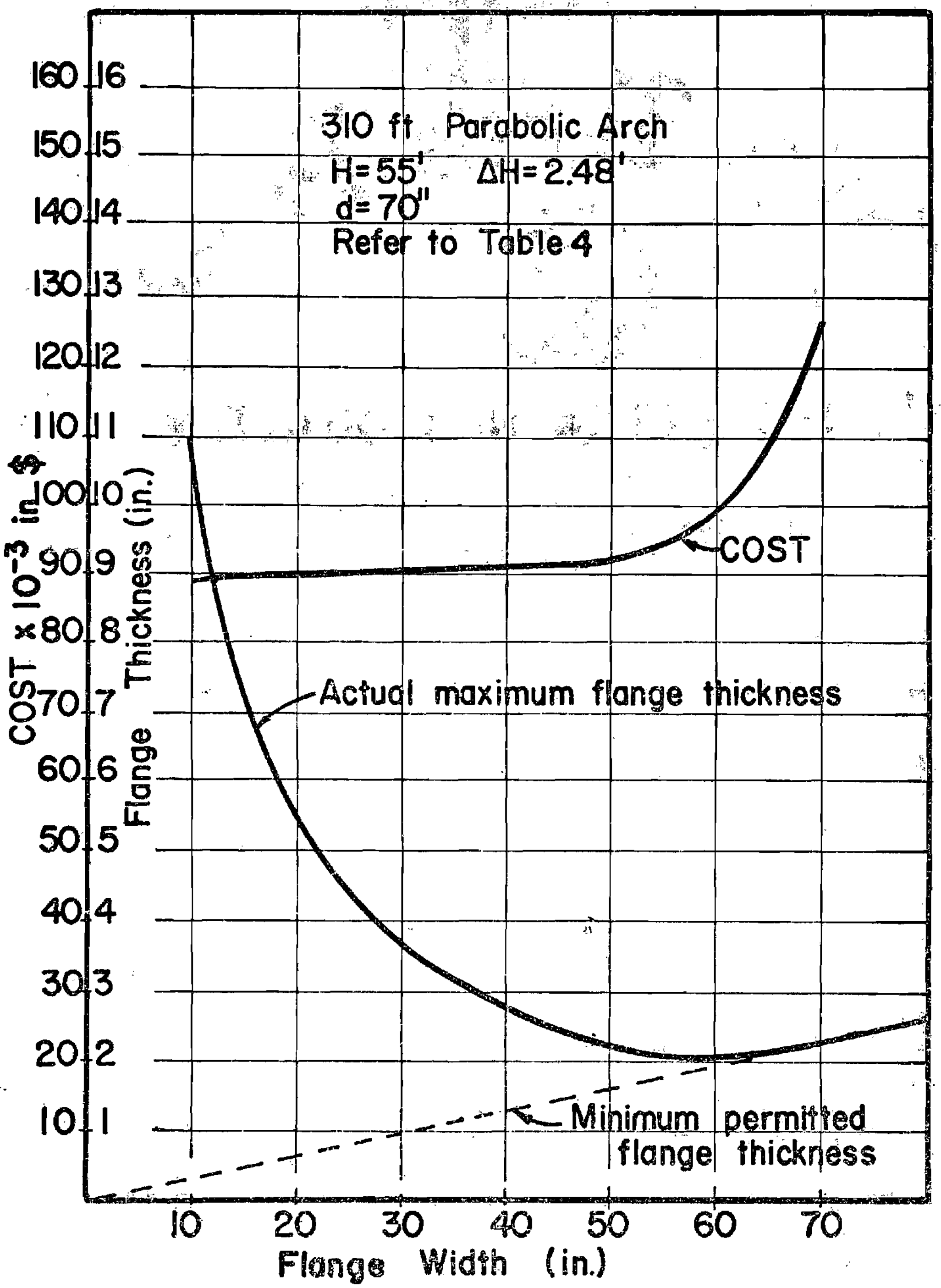


Fig. 31. Influence of Flange Width Variation.

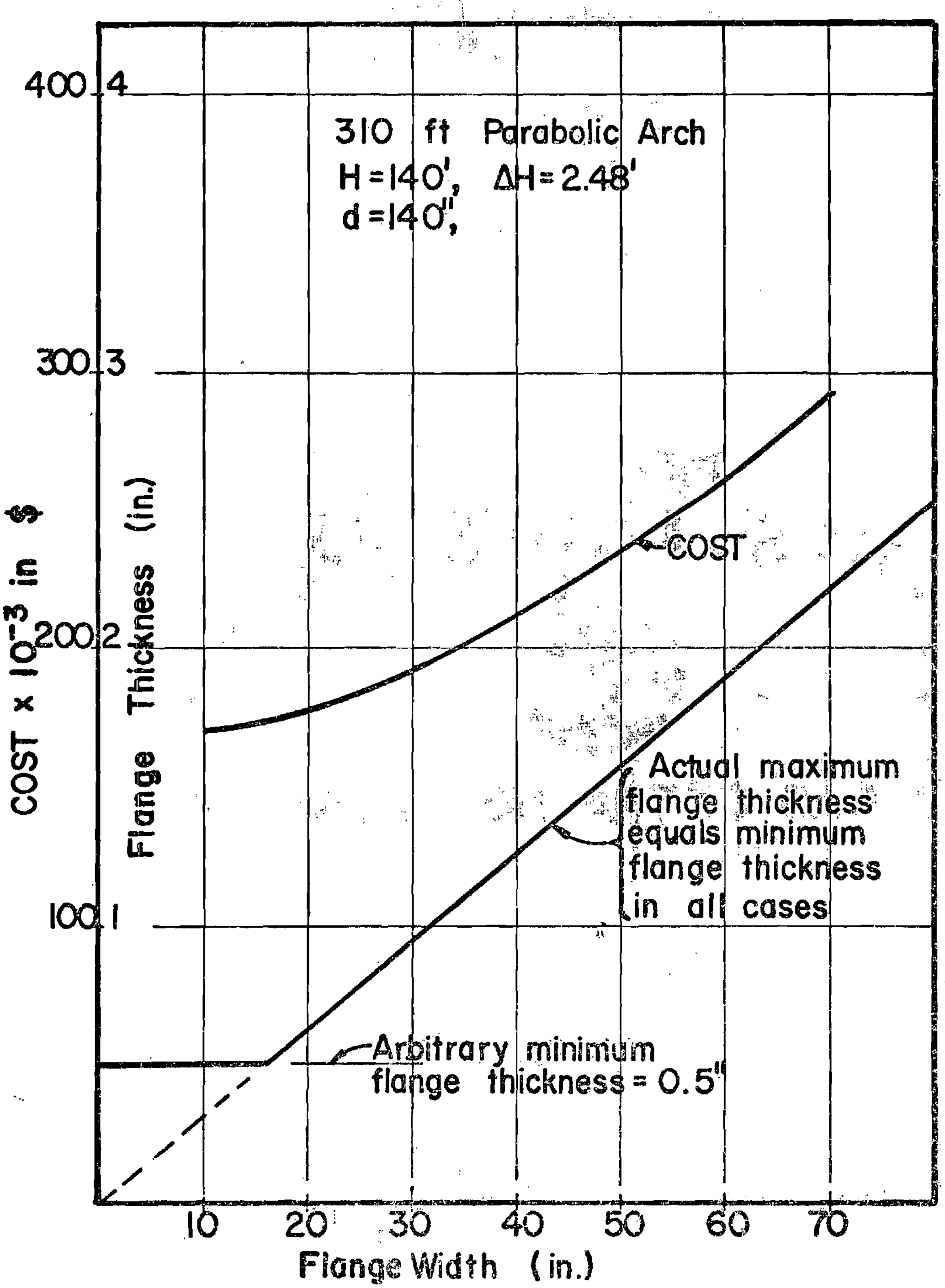


Fig. 32. Influence of Flange Width Variation.

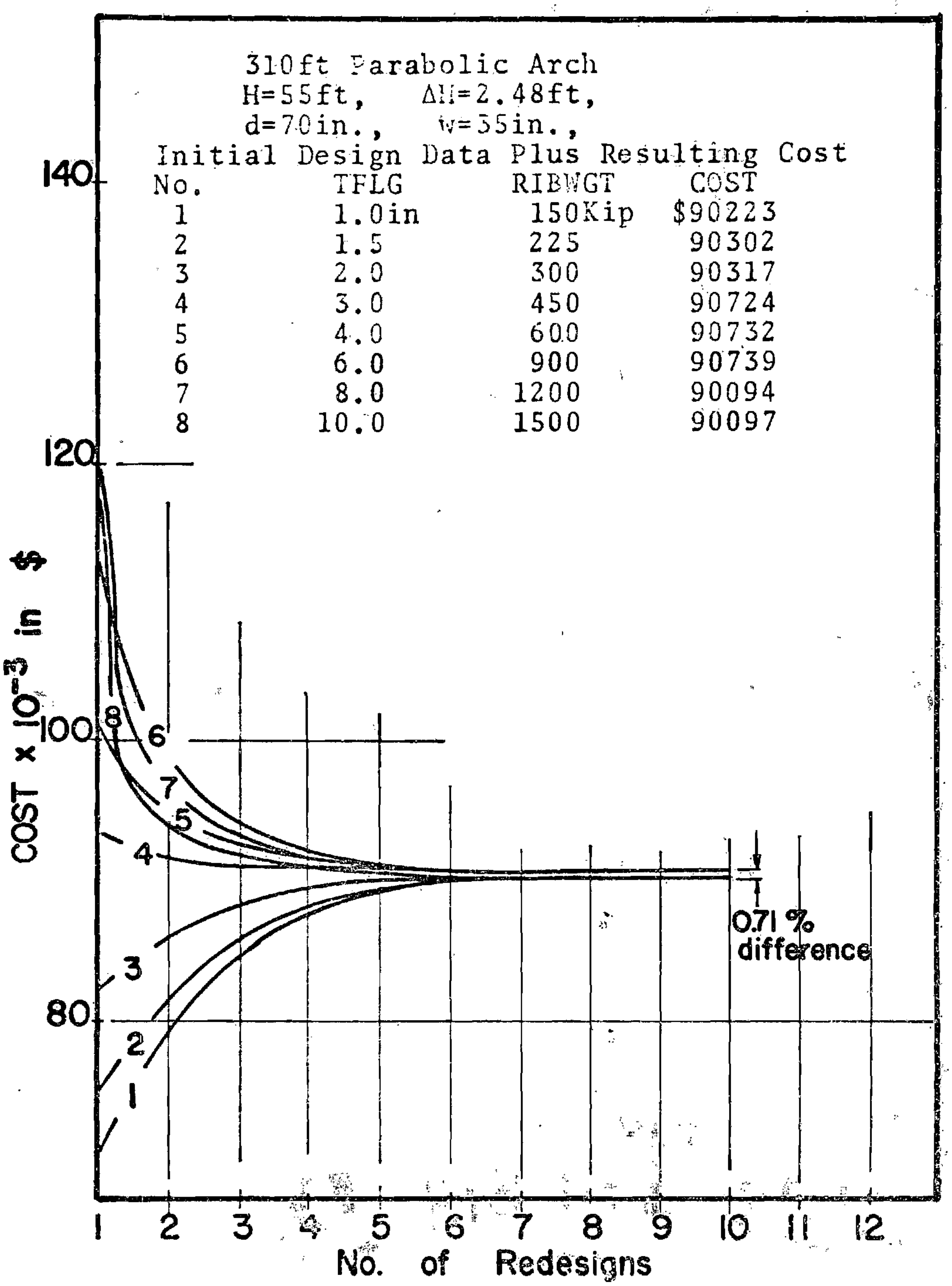


Fig. 33. Reduction of Noise with Redesigns.

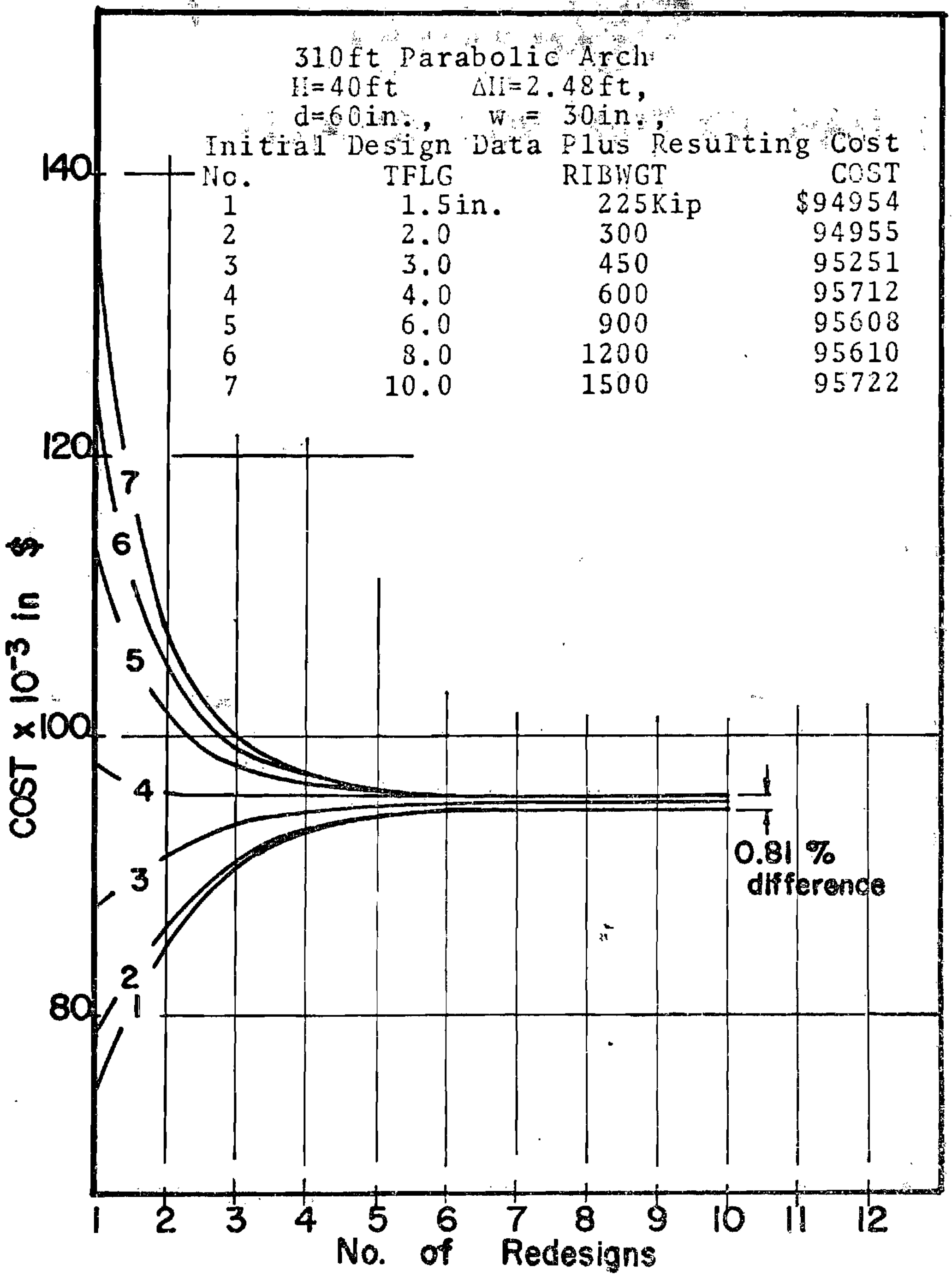


Fig. 34. Reduction of Noise with Redesigns.

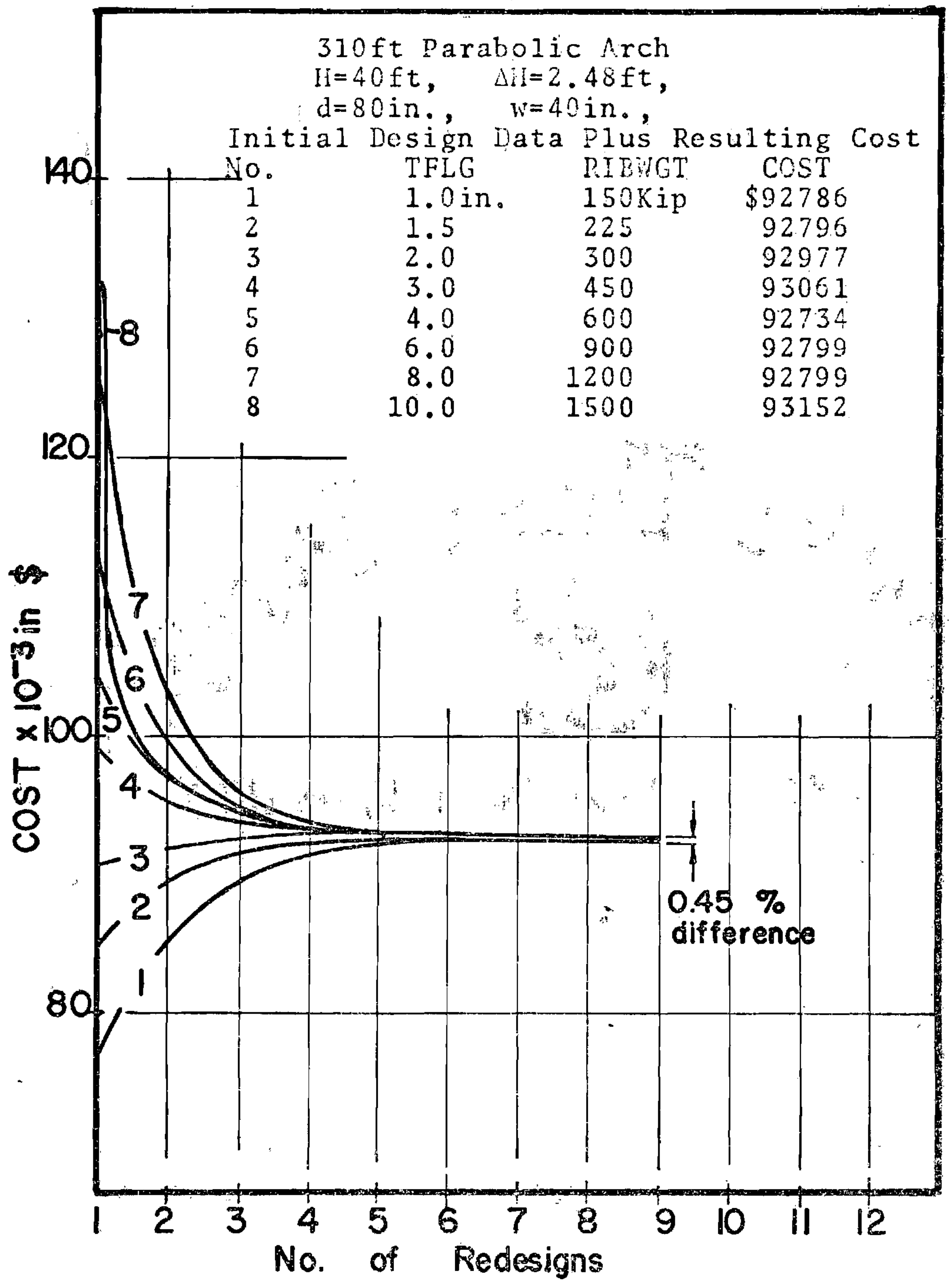


Fig. 35. Reduction of Noise with Redesigns.

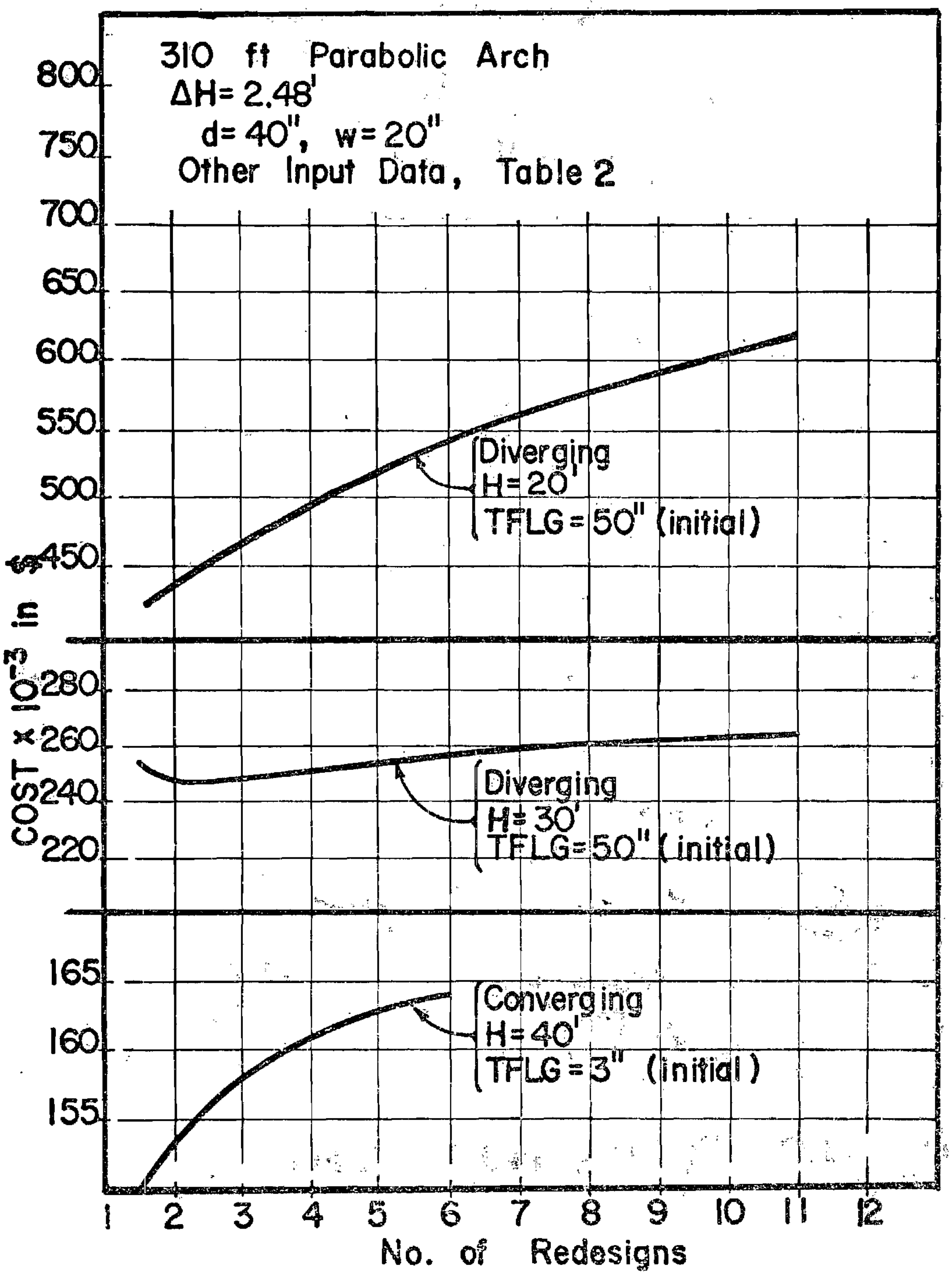


Fig. 36. Divergence with Decreased Arch Height.

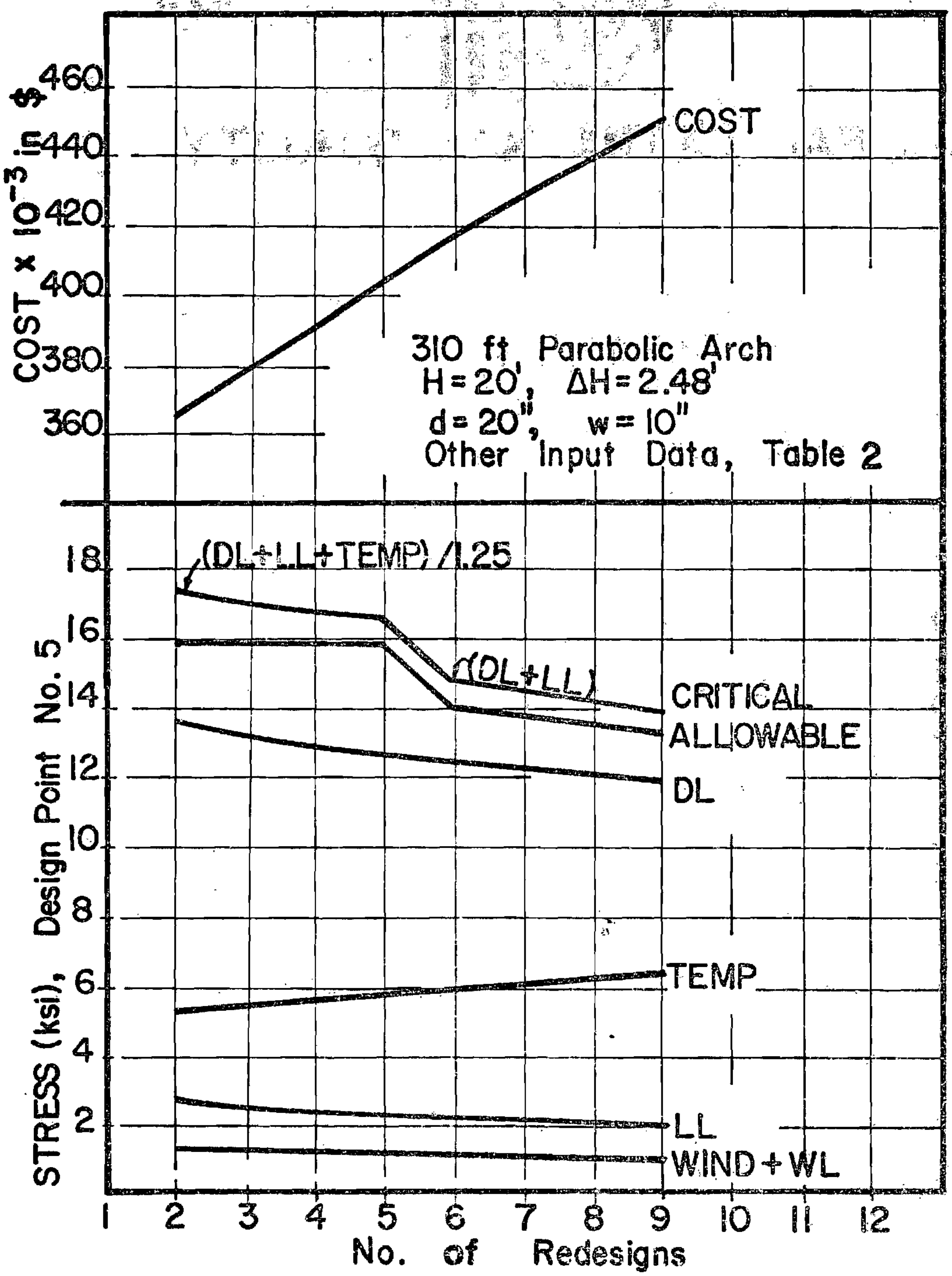


Fig. 37. Relationship of Stresses at Divergence.

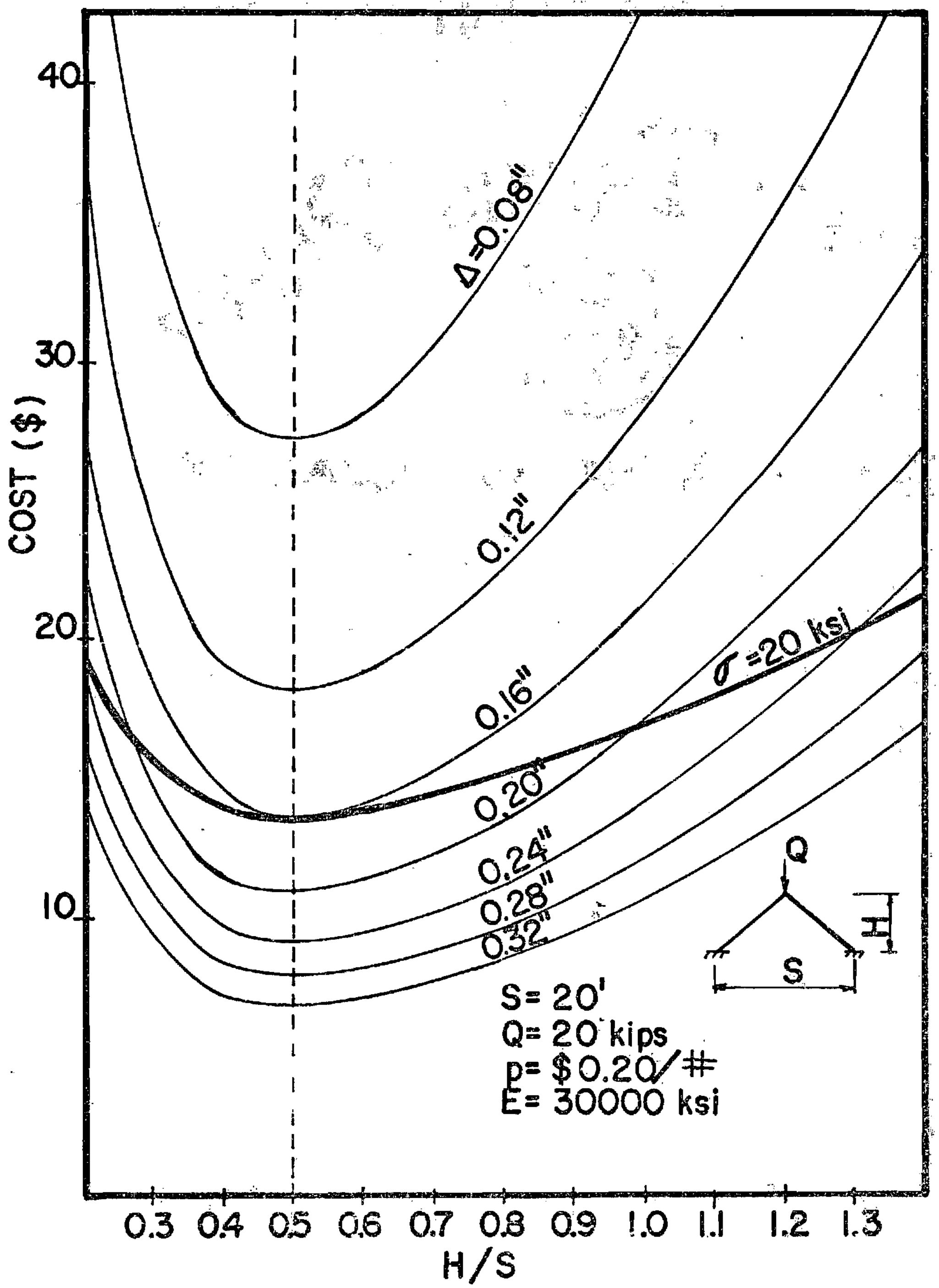


Fig. 38. Stress and Deflection Constraint Curves.

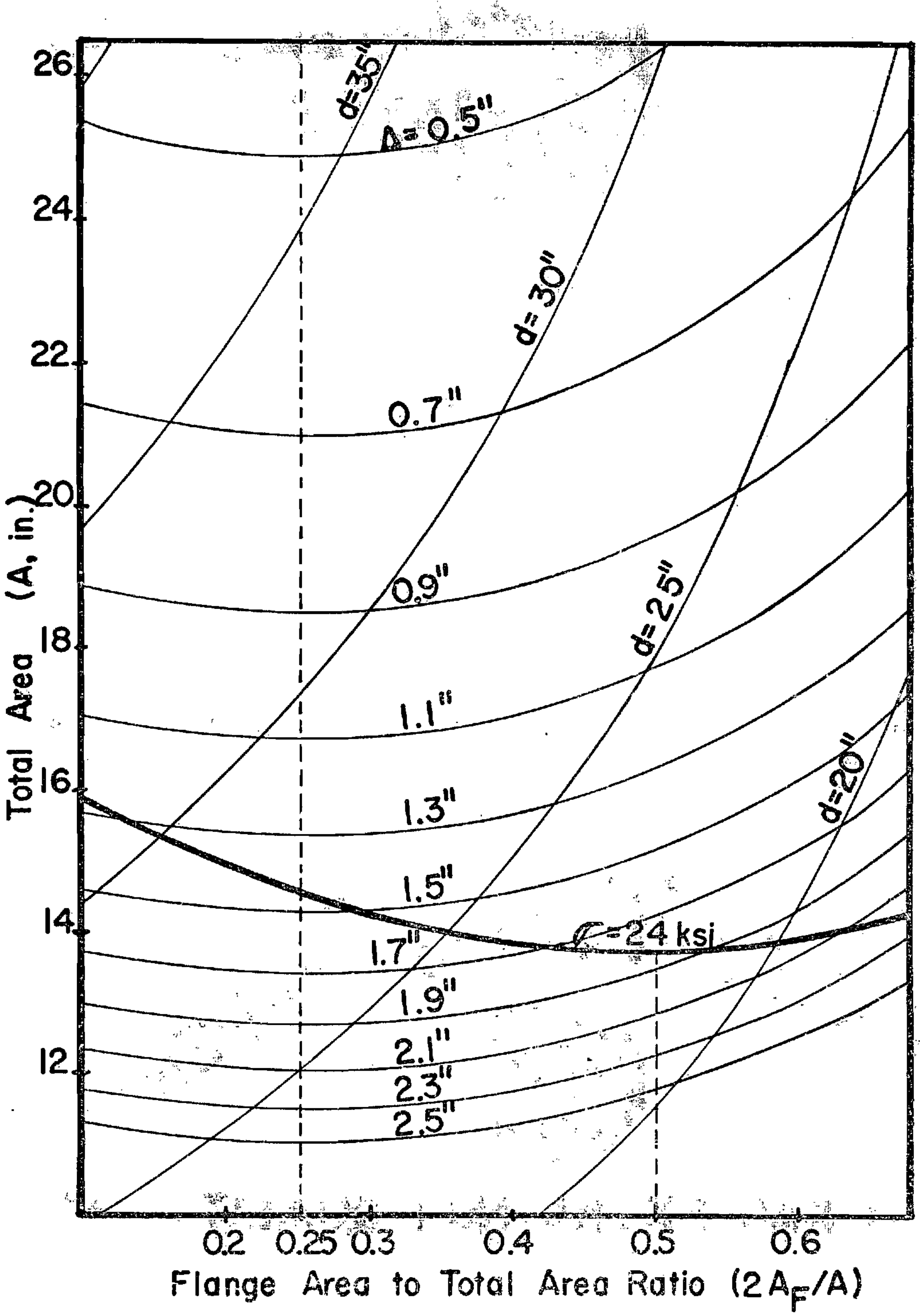


Fig. 39. Stress and Deflection Constraint Curves

for a Beam.

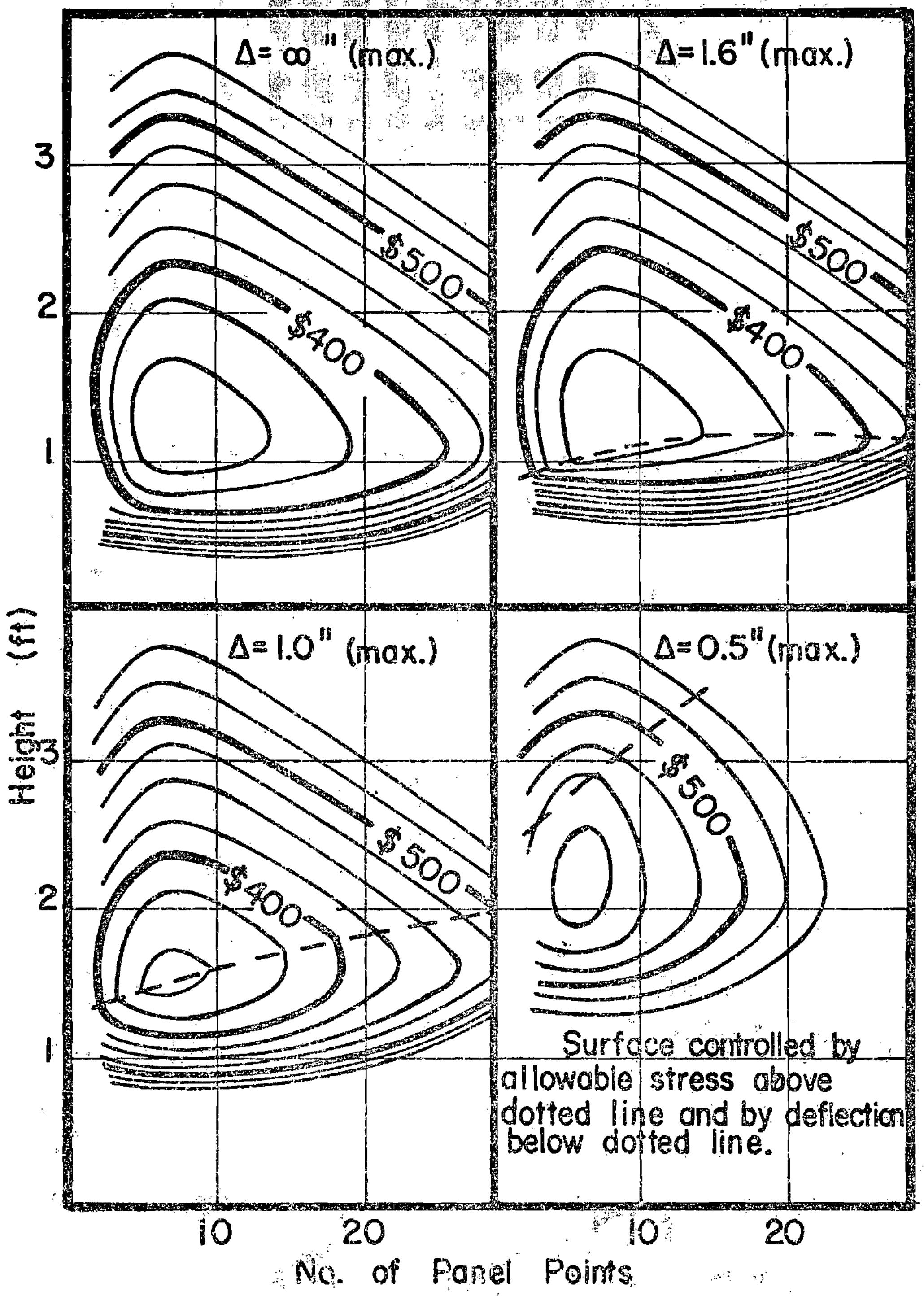


Fig. 40. Stress and Deflection Constraint Contours

for a Bar Joist.

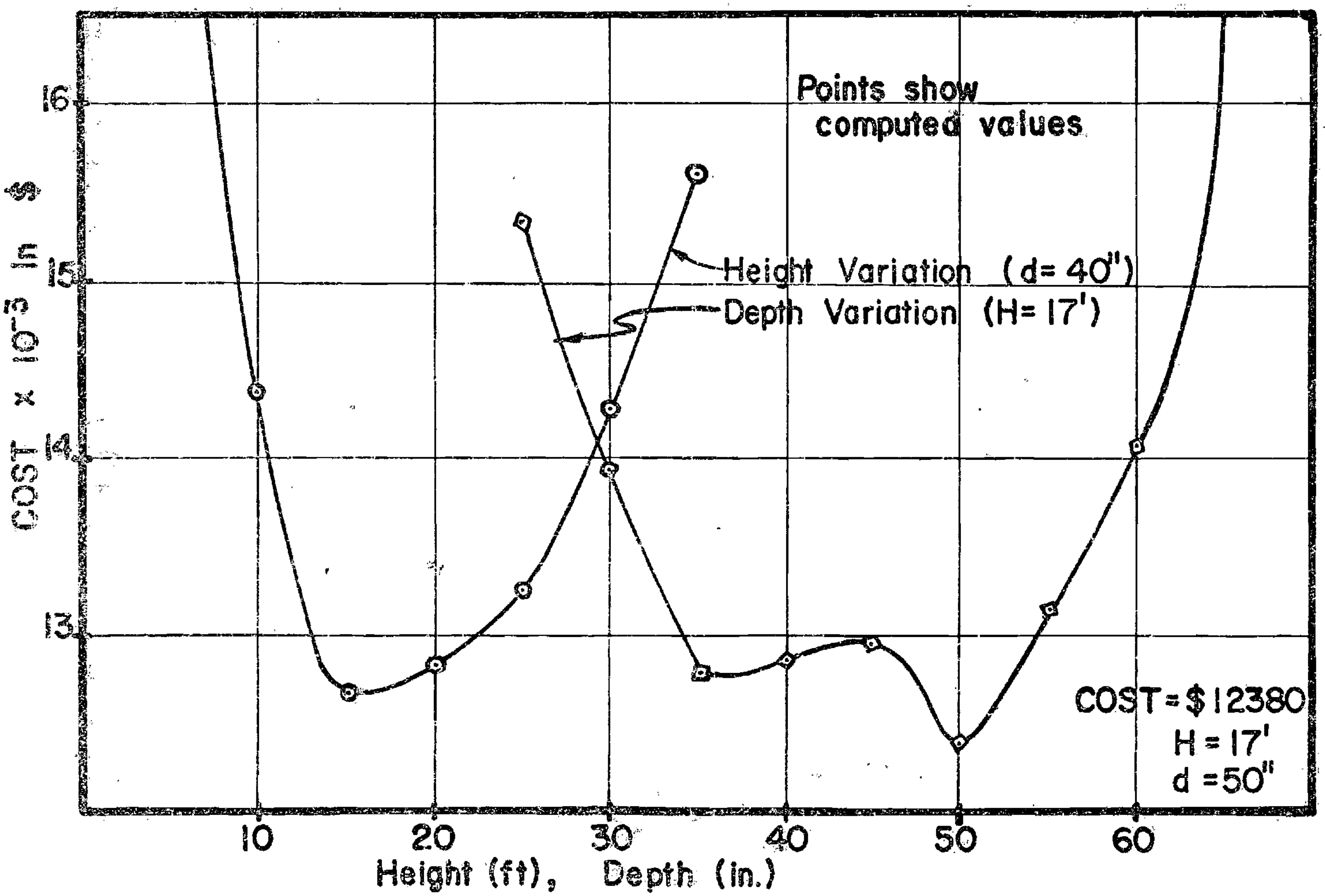


Fig. 41. Equal Interval Search, 100 ft Circular Arch.

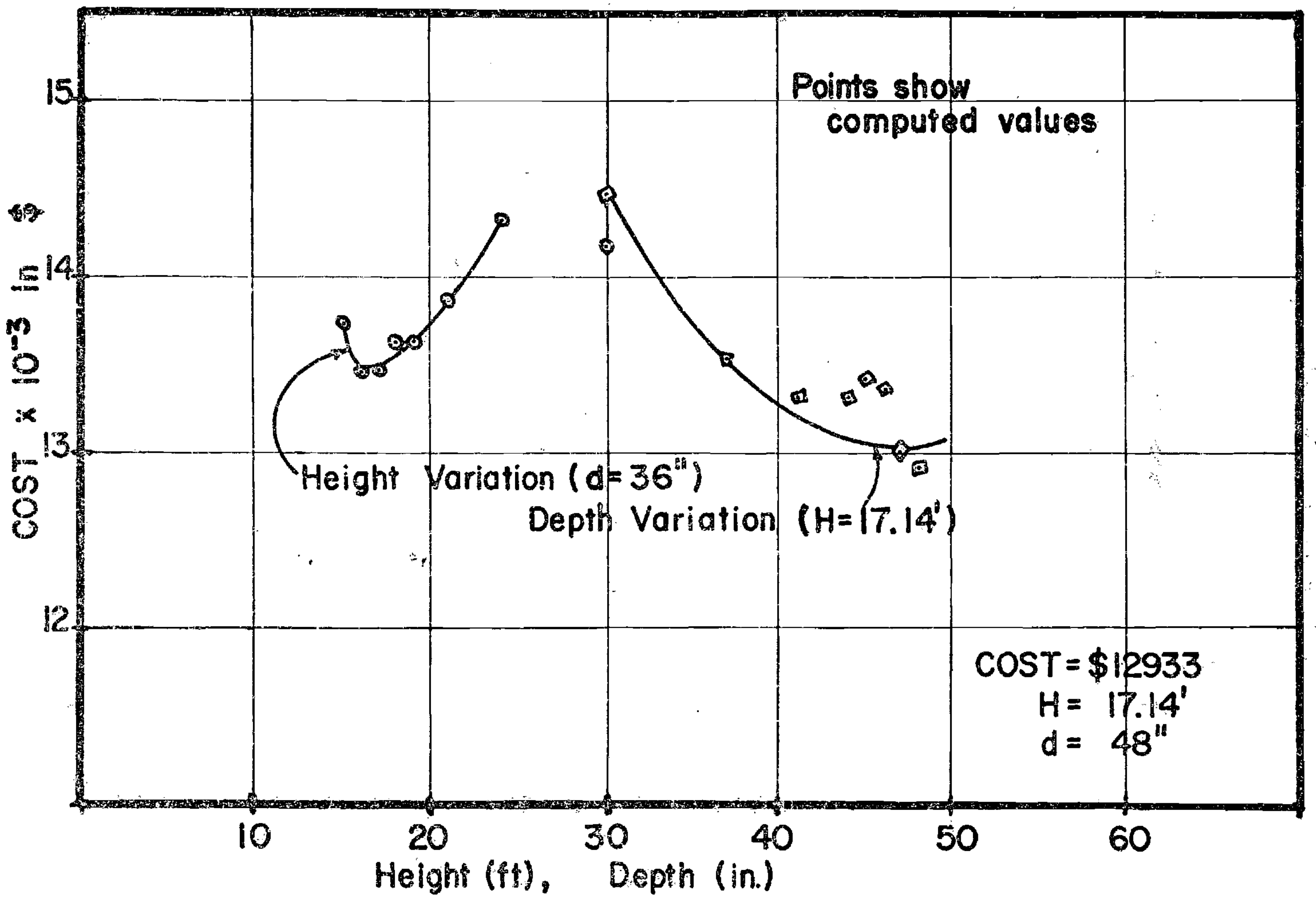


Fig. 42. Fibonacci Search, 100 ft Parabolic Arch.

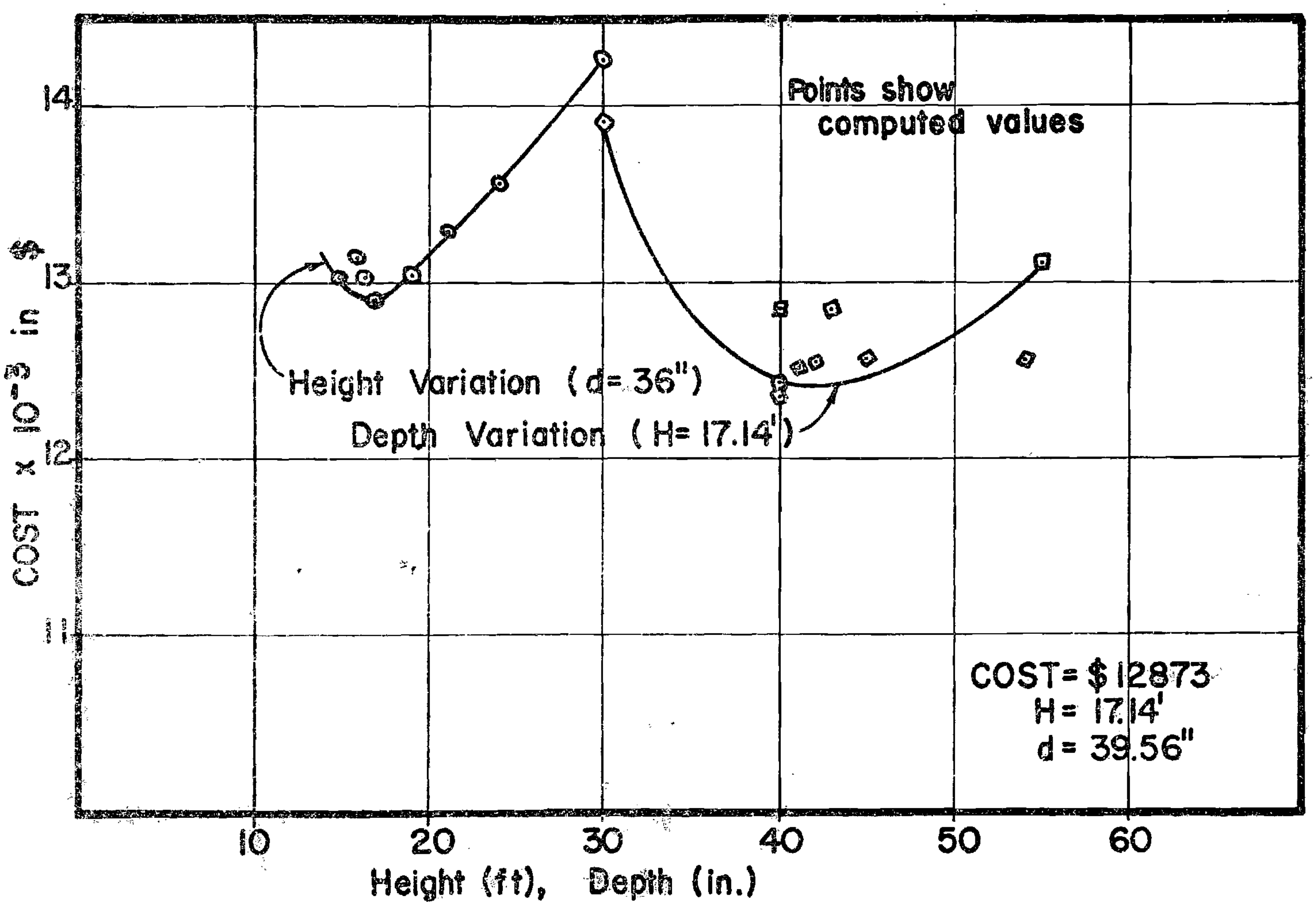


Fig. 43. Fibonacci Search, 100 ft Circular Arch.

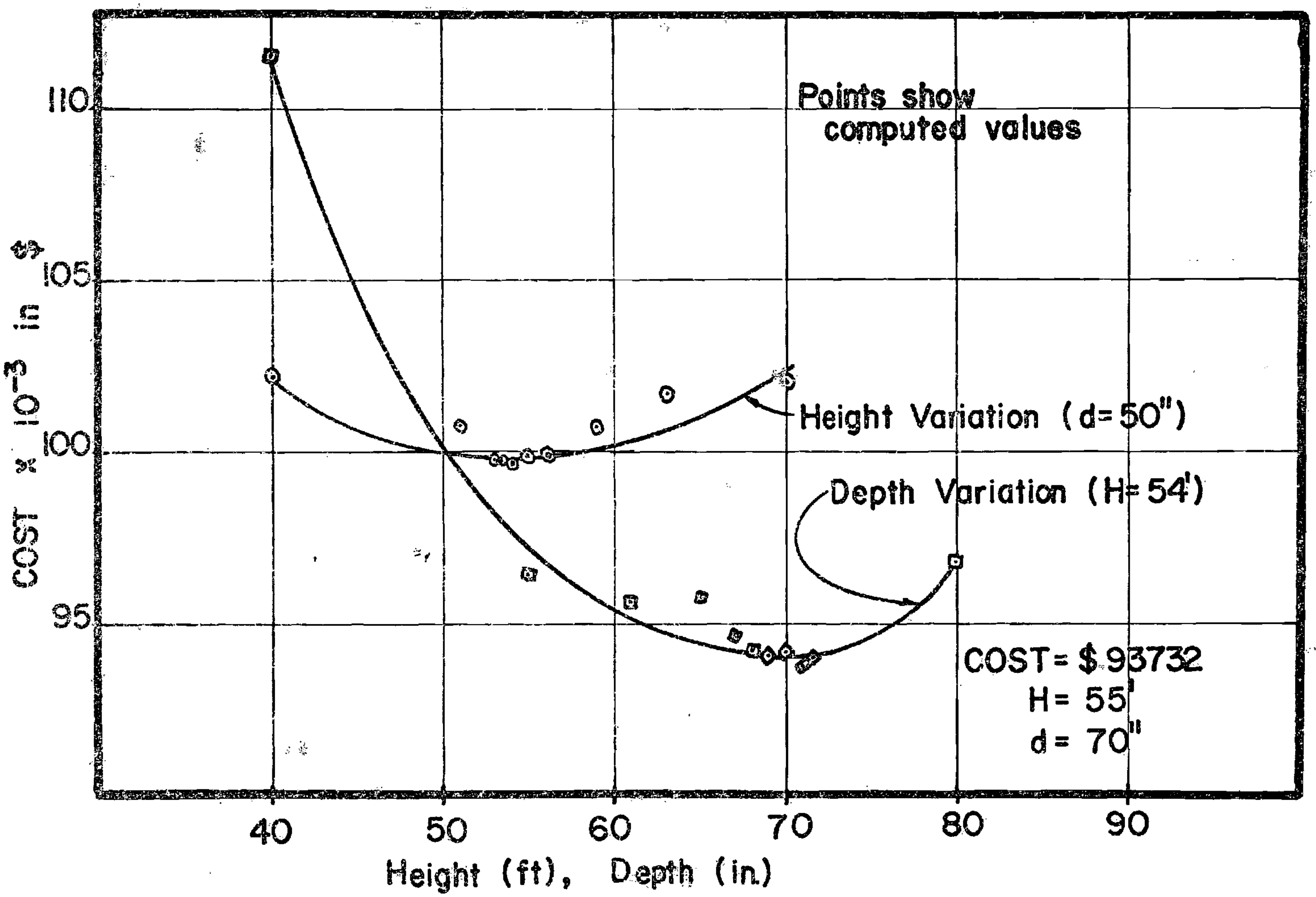


Fig. 44. Fibonacci Search, 310 ft Parabolic Arch.

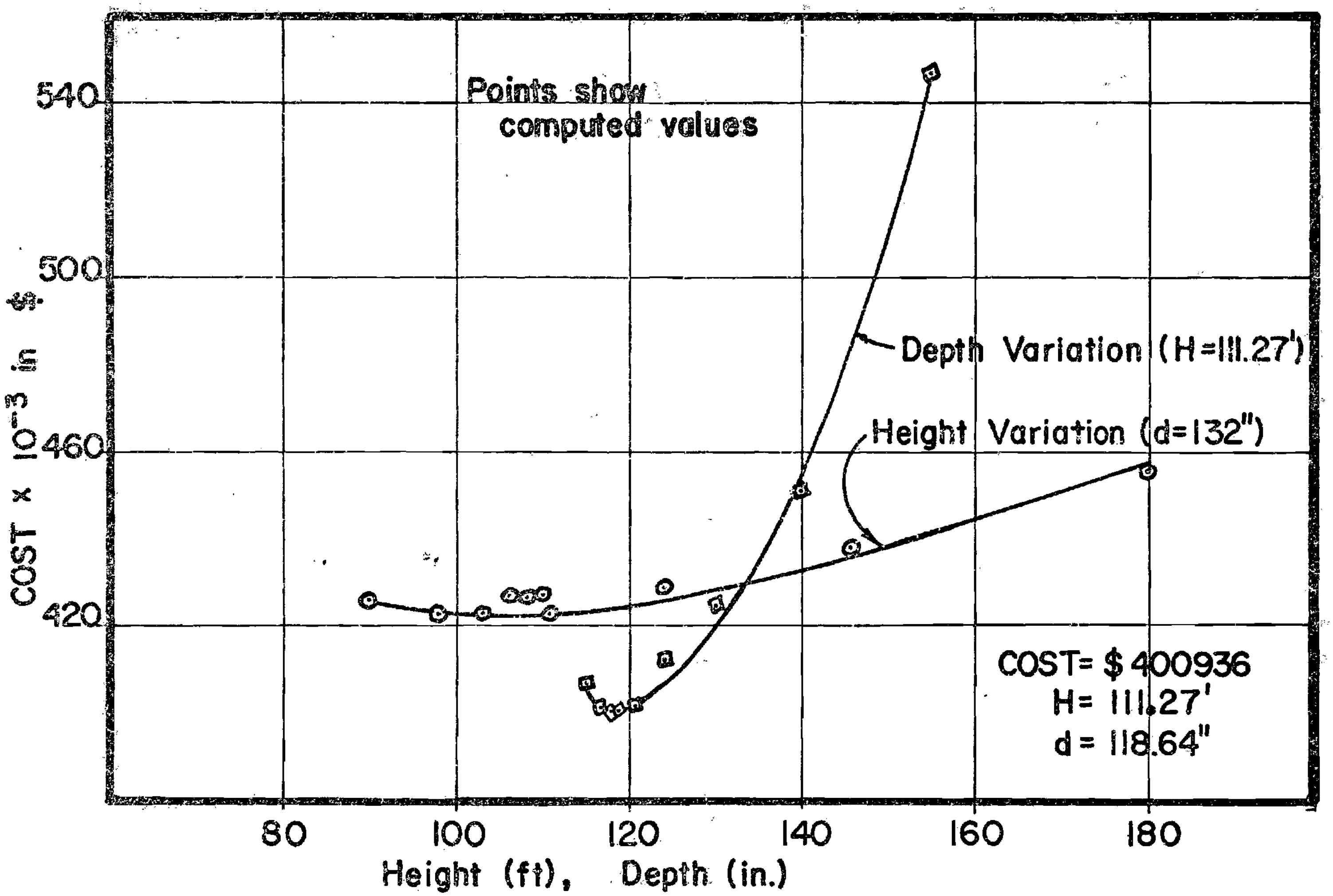


Fig. 45. Fibonacci Search, 600 ft Parabolic Arch.

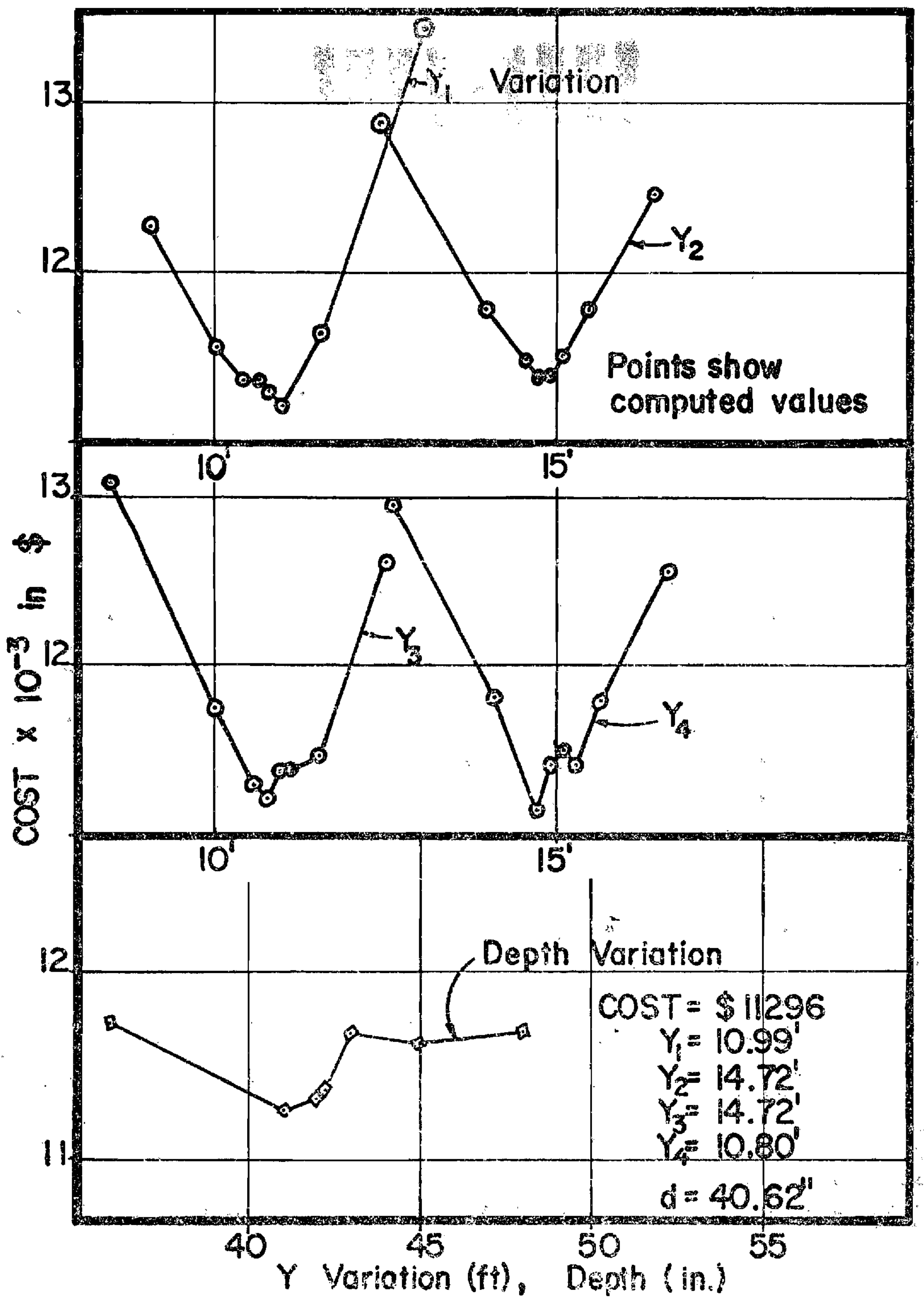


Fig. 46. Fibonacci Search, 100 ft Straight Segmented Arch.

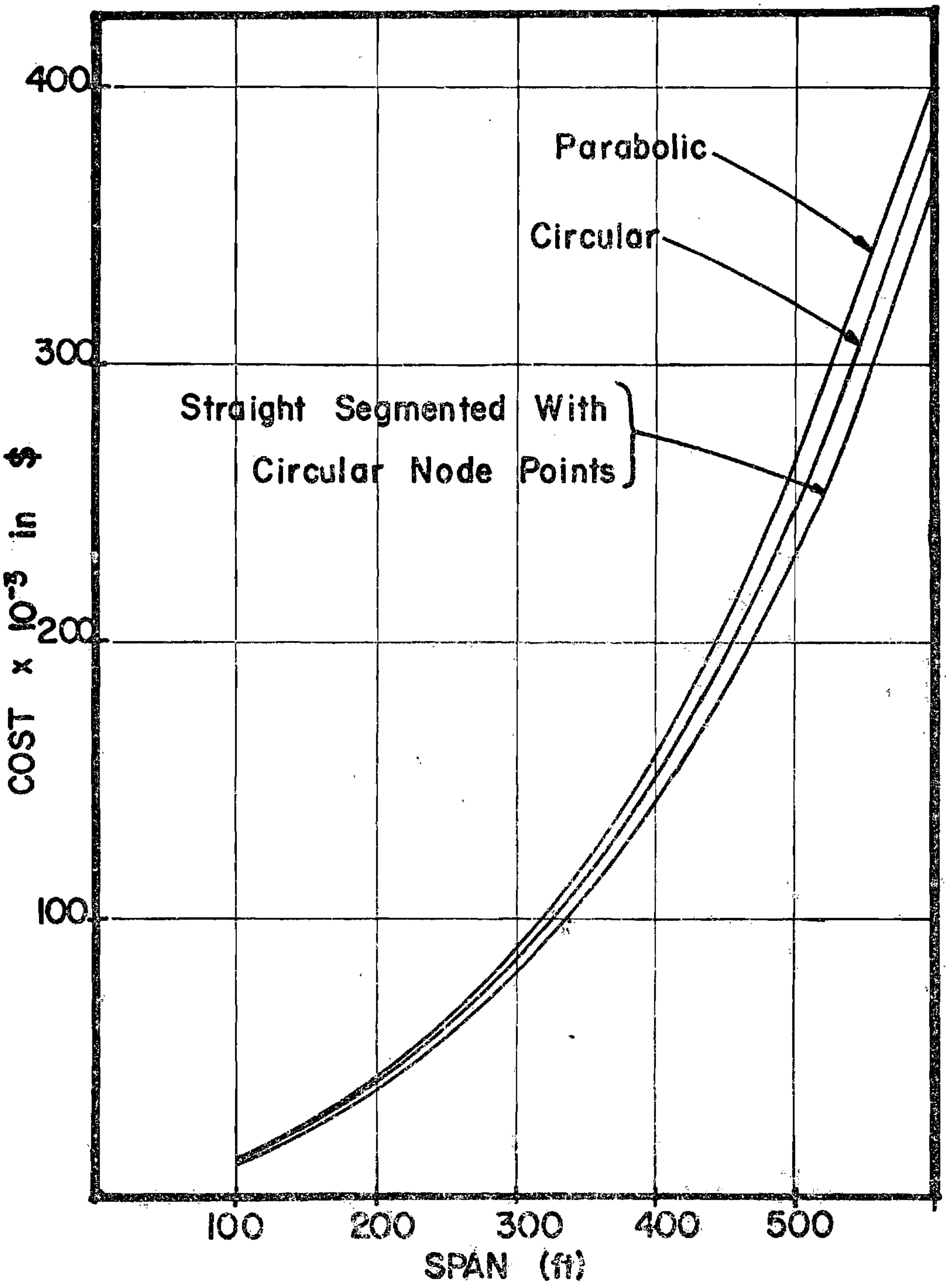


Fig. 47. Fibonacci Search, Results of a Series of Designs, Ratios of H/S and d/S given in Table 6.

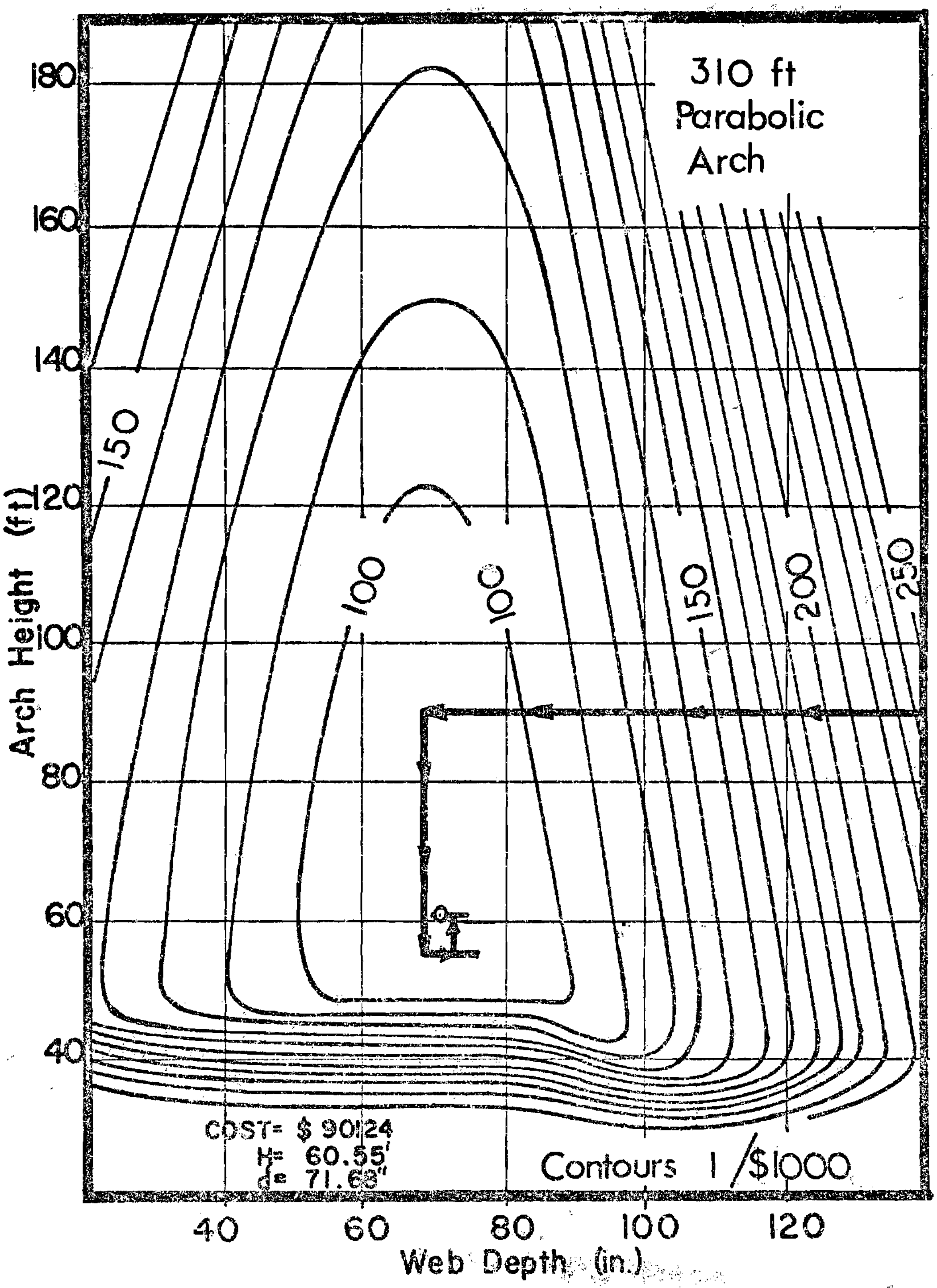


Fig. 48. Univariate Search with Stepping Procedure,

310 ft Parabolic Arch, Path No. 1.

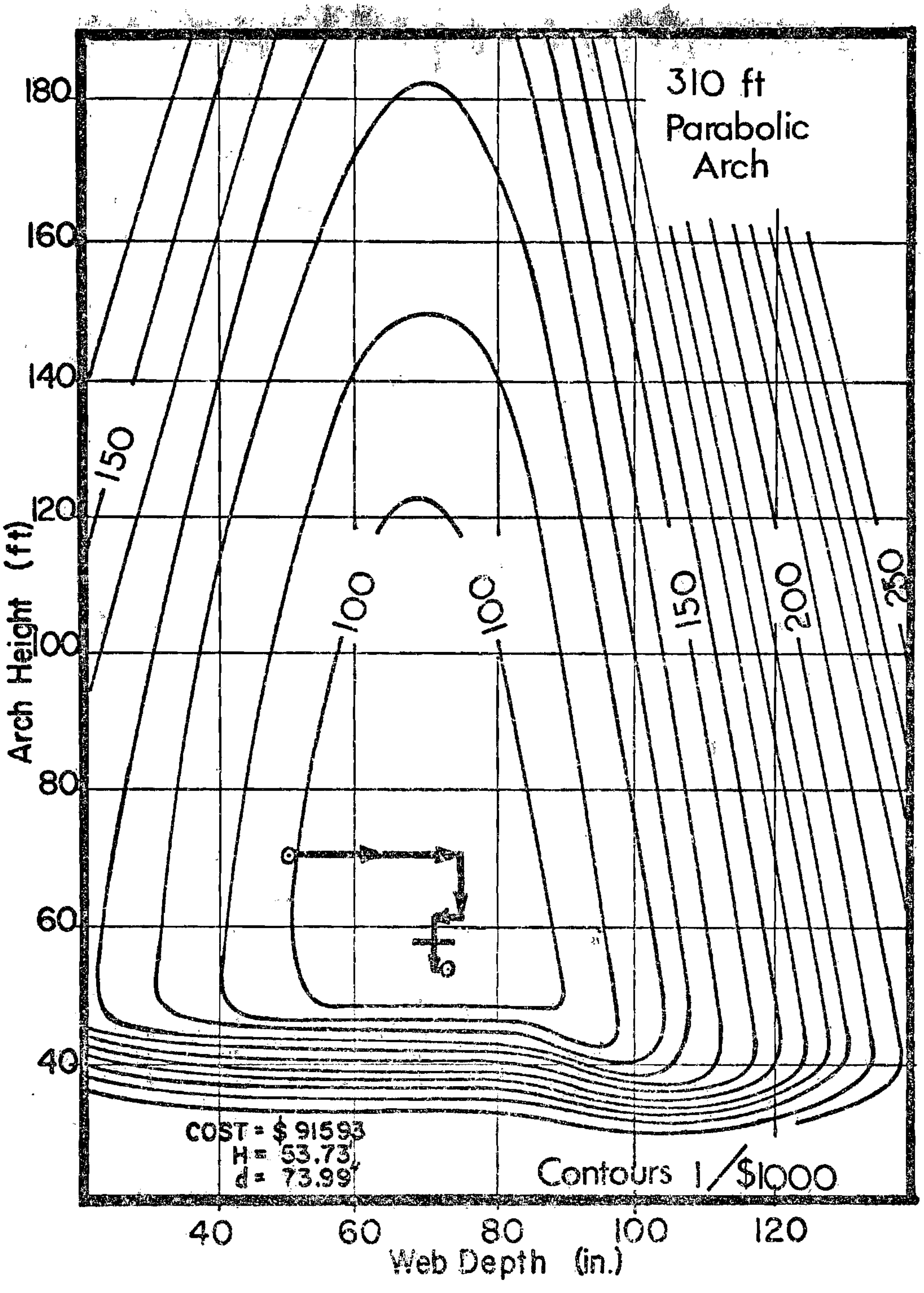


Fig. 49. Univariate Search with Stepping Procedure,

310 ft Parabolic Arch, Path No. 2.

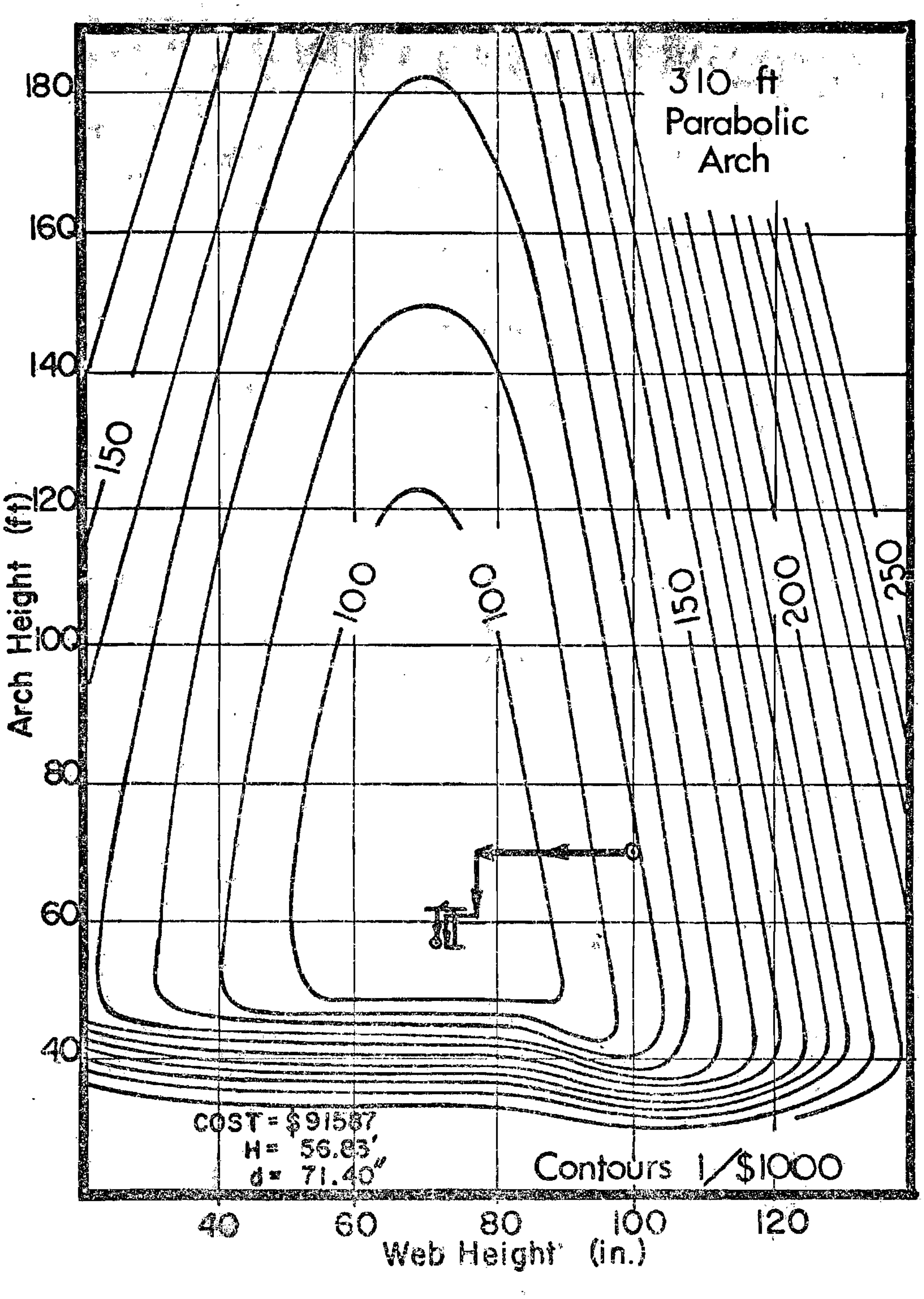


Fig. 50. Univariate Search with Stepping Procedure,

310 ft Parabolic Arch, Path No. 3.

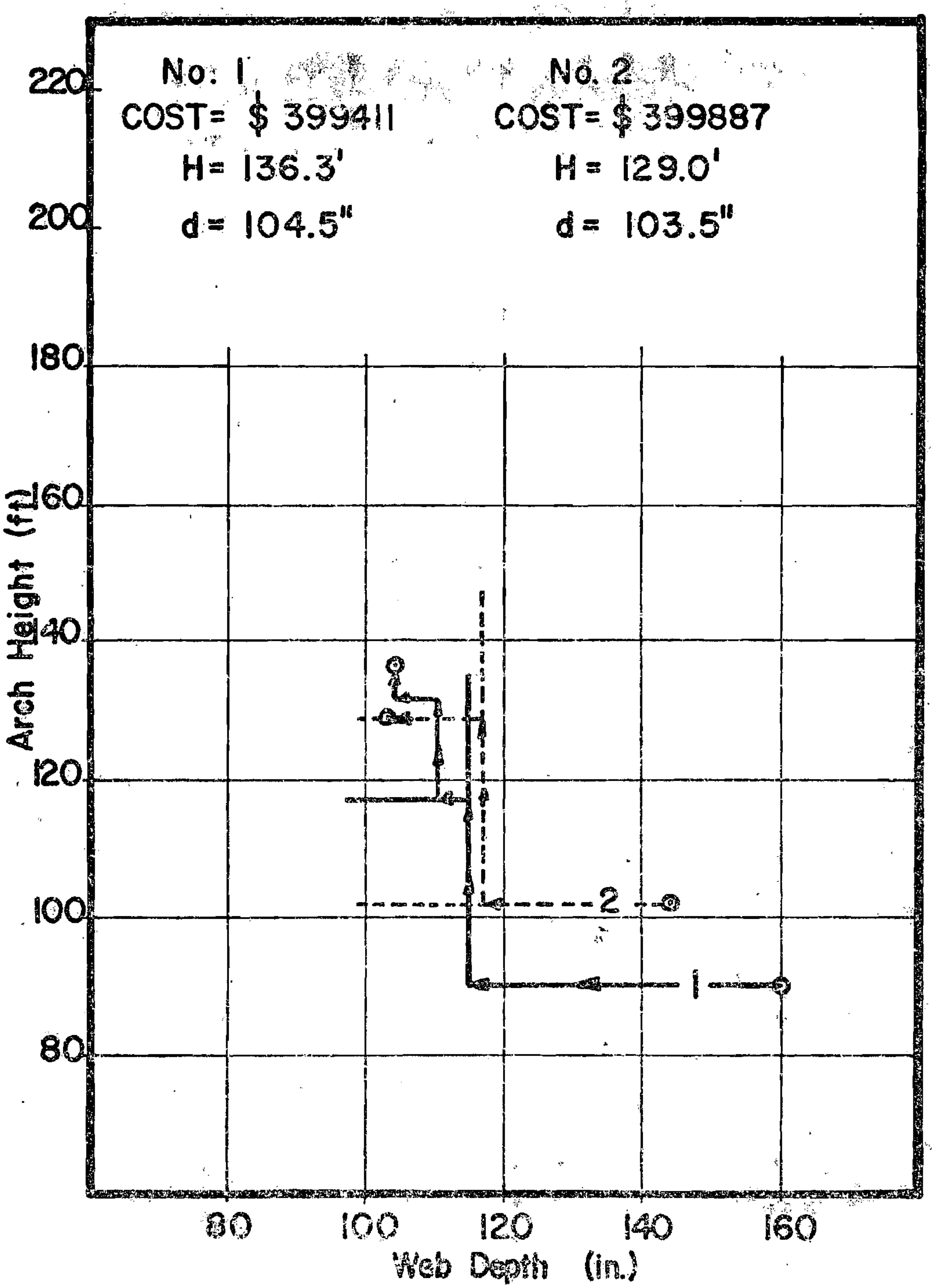


Fig. 51. Univariate Search with Stepping Procedure,

600 ft Parabolic Arch.

APPENDIX C

Glossary of Nomenclature (Thesis Text)

The variables in the thesis text are as follows:

a = height to longitudinal force (LF),

A = area,

$\bar{A}$ = area objective function,

A_F = flange area,

A_o = optimum area,

α = coefficient in fatigue equation,

c = distance from center of gravity to exterior fiber,

CEQ = earthquake coefficient,

COST = magnitude of objective function,

C_1 = constant in arch cost function,

C_w = web depth-to-thickness ratio,

d = depth of web,

D = dead load,

D_o = optimum web depth,

δ = deflection due to a unit load,

Δ = deflection,

ΔH = vertical distance in elevation between the two arch supports,

ΔL = incremental arch length,

Δ_{DL} = deflection due to dead load,

Δ_{LL} = deflection due to live load,

e = eccentricity (M/N),

e_g = largest eccentricity at ends of arc segment between columns,

E = modulus of elasticity,

EQ = earthquake loading,

η = strength constant,

f = function,

f_{ro} = fatigue stress constant,

F_b = allowable stress due to bending,

F_c = column force,

F_r = reduced allowable stress due to fatigue,

F_s = reduced allowable stress due to column action,

F_u = ultimate strength of steel,

F_Y = yield strength of steel,

g = constraint equation,

γ = coefficient of temperature expansion,

H = height of arch,

H_A, H_B = horizontal reaction components,

I = moment of inertia,

I_m = impact coefficient,

k = geometry constant for parabolic arch,

k_c = X_L/S,

k_1 = fatigue stress constant,

K = stiffness constant for deflection calculations,

L = arc length,

LF = longitudinal force,

LL = live load

L_c = arc length between columns,

λ = Lagrange's multiplier,

m = moment due to a unit load,

M_w = number of webs,

M = moment,

M_A = moment due to wind about point A,

M_C = moment due to wind about point C,

n = thrust due to a unit load,

N = thrust,

p, p_1, p_2 = price,

P_I = interatate load

P_L = lane concentrated load for bending,

P_{T1}, P_{T2} = truck load,

ϕ = variable used in computing allowable stress,

q = uniform load,

Q = concentrated load,

r = radius of gyration,

R = radius of curvature,

R_f = ratio of minimum stress to maximum stress for fatigue calculation,

ρ = density,

S = span,

S_L = large span,

SQ = variable for parabolic length equation,

S_s = small span,

S_R = horizontal distance between arch ribs,

σ = stress,

t = temperature change,

t_f = flange thickness,

t_w = web thickness,

T = value for step function,

TD = temperature drop,

TEMP = force caused by a temperature change,

TR = temperature rise,

θ = slope of arch rib,

V = shear,

V_A, V_B = vertical reaction components,

w = flange width,

WIND = force caused by wind on structure,

WL = force caused by wind of fictitious live load,

W_L = uniform lane load,

W_S = uniform sidewalk live load,

x = horizontal distance from left support,

X_L = distance to load,

Y = vertical distance to arc from left support.

APPENDIX D

Glossary of Nomenclature (Computer Program)

The variables which are used in the computer program are as follows:

A = temporary area,

ALFA = temperature coefficient, coefficient in fatigue equation,

ALFAK = initial step size,

ALLOWABLE = allowable stress,

AREA = cross sectional area of rib, area under influence line,

ASTR = temporary allowable stress,

ATEMP = temporary area,

AVES = average stress used for finding the allowable,

C = cost for improved search steps, distance from center of gravity to the exterior fiber, temporary earthquake coefficient, temporary cosine,

CEQ = earthquake coefficient,

CF = file in identifier,

CI = cost for all search steps,

CL1, CL2, CL3, CL4 = labels for conjugate direction search,

CONJ = label to start conjugate direction search,

CONV = label to start the final design,

COSE = cosine of angles THED,

COST = total cost of the arch rib,

CRITM = governing moment,

CRITN = governing thrust,

CRITSTR = governing stress,

CRITV = governing shear,

CYCLES = number of cycles for fatigue stress computations,

D = web depth for improved search steps, temporary rib depth, temporary deflection,

D1 = temporary maximum live load deflection,

D2 = temporary minimum live load deflection,

DECKD = deck depth,

DEF = temporary deflection,

DEFL = deflection influence lines,

DEFLECTION = deflection for improved search steps,

DEFLI = deflection for all search steps,

DEFM = deflection due to moment,

DEFN = deflection due to thrust,

DEL = difference between governing and allowable stress,

DELC = length of arc segment between columns divided by 10,

DELH = vertical distance between high support and low support for an unsymmetrical arch,

DELS = increment of arc length,

DELSTR = difference between governing and allowable stress,

DEPTH = web depth,

DES = search decision variable,

DES5 = a counter that limits the number of designs to 5 per H or d change,

DESIGN = number of improved designs required to find the least cost design,

DESIGNS = total number of designs required to find the least cost design,

DESN = search decision variable,

DI = web depth for all search steps,

DIST = horizontal distance between column points,

DLD = dead load deflection,

DLM = dead load moment,

DLN = dead load thrust,

LDV = dead load shear,

DOPT = resulting web depth,

DUMMY = dummy and temporary variable,

E = modulus of elasticity,

EG = large eccentricity at the end of the arch rib segment,

EPS = tolerance used in geometry program, stress tolerance used in adjustment program,

EQM = earthquake moment,

EQN = earthquake thrust,

EQS = earthquake shear,

ES = small eccentricity at the end of the arch rib segment,

ETA = strength constant,

F = length function for the parabolic arch, temporary column force, concentrated live load plus impact,

F1 = derivative of the length function, derivative of stress function, F1, F2 = temporary truck live load plus impact,

FAVE = temporary average stress,

FC = deck column force,

FCl = temporary live load column force,

FCLL = live load column force,

FCSW = column force due to sidewalk live load,

FFC = temporary column force,

FINISH = label for finish of program,

FMTi (i = 1, . . . 36) = format identifiers,

FRO = stress value for fatigue stress computations,

FU = ultimate strength of steel,

FY = yield strength of the steel,

FYOPT = decision variable to decide when deflections should be computed,

FYY = temporary yield strength of the steel,

GD = normalized movement in depth direction,

GH = normalized movement in height direction,

GK = unit movement for step k,

GOMIN = variable that directs the search to either the conjugate direction or the ravine search,

GOO = variable which determines whether stress, shear or deflection is to be computed in the interstate, truck and lane subprogram,

GOSHEARCH = variable that directs the search within the conjugate direction and ravine search,

H = height for improved search steps, height of arch, constant used in Simpson's numerical integration, H1, H2 = height to longitudinal force at columns L1 and L2,

HC = height to center of mass of the deck,

HCG = height to center of mass of the deck,

HD = height to the deck surface,

HDECK = height to the deck surface,

HEIGHT = height of the arch,

HI = height for all search steps,

HLEFT = horizontal reaction component at left support,

HLF = height to longitudinal force,

HOPT = resulting height,

HORIZD = horizontal distance between the two arch ribs,

HORIZS = horizontal reaction component influence line,

HRIGHT = horizintal reaction component at right support,

I = counting variable, temporary moment of inertia,

IDEFL = live load deflection,

II = counting variable,

IMPACT = live load impact coefficient,

INER = temporary moment of inertia,

INERTIA = moment of inertia,

IR = temporary moment of inertia,

ITEMP = temporary moment of inertia,

J = counting variable,

JJ = counting variable,

K = constant for geometry computions, counting and limiting variable,

Kl = constant for fatigue stress computations,

L = limit integer used in several places,

L1, L2 = column points at which the longitudinal force may be transferred,

L1, L2, L3, L4 = computer statement labels,

LAB1, LAB2, LAB3 = computer statement labels,

LC, LCC = number of improved designs between direction changes for the conjugate direction search,

LDEFL = maximum live load deflection,

LENGTH = arc length in geometry program,

LENGTH1 = temporary arc length in geometry program,

LFM = longitudinal force moment,

LFN = longitudinal force thrust,

LFV = longitudinal force shear,

LGTH = total length of arch rib, length of arc between columns = 10 x DELC,

LMAXD = maximum live load deflection,

LMAXM = maximum live load moment,

LMAXN = maximum live load thrust,

LMAXS = maximum live load stress,

LMAXV = maximum live load shear,

LMIND = minimum live load deflection,

LMINM = minimum live load moment,

LMINN = minimum live load thrust,

LMINS = minimum live load stress,

LMINV = minimum live load shear,

LONGF = longitudinal force,

LR = number of improved designs between direction changes for the ravine search,

M = temporary moment,

M1 = temporary maximum live load moment,

M2 = temporary minimum live load moment,

MAXDEFL = maximum live load deflection at column points,

MAXM = temporary maximum moment,

MAXMOM = temporary maximum live load moment,

MAXN = temporary maximum thrust,

MAXS = temporary maximum stress,

MAXSHEAR = temporary maximum live load shear,

MAXSTRESS = temporary maximum live load stress,

MAXSWM = maximum sidewalk live load moment,

MAXSWN = maximum sidewalk live load thrust,

MAXSWV = maximum sidewalk live load shear,

MAXTHRUST = temporary maximum live load thrust,

MAXV = temporary maximum shear,

MD = moment due to temperature drop,

MG = large moment at end arch rib segment,

MI = moment due to unit load at point i,

MINDEFL = minimum live load deflection at column points,

MINM = temporary minimum moment,

MINMOM = temporary minimum live load moment,

MINN = temporary minimum thrust,

MINS = temporary minimum stress,

MINSHEAR = temporary minimum live load shear,

MINSTRESS = temporary minimum live load stress,

MINSWM = minimum sidewalk live load moment,

MINSWN = minimum sidewalk live load thrust,

MINSWV = minimum sidewalk live load shear,

MINTHRUST = temporary minimum live load thrust,

MJ = moment due to unit load at point j,

MM = moments MI and MJ multiplied together,

MOM = moment influence lines,

MR = moment due to temperature rise,

MS = small moment at end of arch rib segment,

N = temporary thrust, temporary upper limit,

N1, N2 = upper and lower limit for numerical integration according to Simpson's rule,

N1 = temporary maximum live load thrust, temporary NC-1,

N2 = temporary minimum live load thrust,

NC = number of columns,

NC1 = number of column points minus one,

ND = 10 x NP = number of segments that the arch rib is divided into for numerical integration purposes, thrust due to temperature drop,

NEG = sign change variable,

NG = reduction in basic step size counter,

NI = thrust due to a unit load at point i,

NJ = thrust due to a unit load at point j,

NN = thrusts NI and NJ multiplied together,

NO, NO1 = search met with failure, start decelerating step size,

NP = number of design points,

NP1 = number of design points minus one,

NR = thrust due to temperature rise,

NRIDGE = exit from ravine search variable,

P = temporary concentrated live load,

P1, P2 = temporary truck live load,

PF = file out identifier,

PHI = temporary slope angle, constant for fatigue stress computations,

PI = interstate live load,

PL = concentrated lane load,

PRICE1 = price of steel for webs and flanges,

PRICE2 = price of steel for stiffeners and diaphragms,

PT1, PT2 = truck live load,

R = weight of flanges and webs for improved search steps, ratio of minimum stress to maximum stress, temporary radius,

R1, R2, R3, R4, R5, R6, R7, R8 = temporary ratio values for computing the maximum and minimum live load forces due to truck and interstate loading,

RAD = radius of gyration,

RADIUS = radius for circular arch,

RATIO = ratio of the web depth to the flange width,

RD1, RD2, RD3 = labels for the ravine search,

RI = weight of flanges and webs for all search steps,

RIBWGT = total weight of the rib flanges and webs,

RIDGE = label to start the ravine search,

RL1, RL2, RL3, RL4, RL5, RL7, RL8, RL9 = labels for ravine search,

S = weight of diaphragms and stiffeners for improved search steps, temporary stress, temporary shear, temporary sine,

S1 = temporary maximum live load stress,

S2 = temporary minimum live load stress,

SHEAR = shear influence lines,

SI = weight of diaphragms and stiffeners for all search steps,

SINE = sine of angles THED,

SPAN = arch span,

SPANL = large span,

SPANS = small span,

SQ = square root value,

START = label at the start of the read statements and program,

STEPK = search step size,

STIFFENERS = weight of the stiffeners and diaphragms,

STR = temporary stress,

STRESS = stress influence lines,

SUM = several temporary sum values,

SUM1, SUM2, SUM3 = totals of Simpson integration for the influence lines for the reaction components,

SUM4, SUM5 = totals of Simpson integration for reaction components caused by the longitudinal force,

SUMTEMP = temporary summation,

SW = computer switch statement,

TARCH = type of arch,

TD = temperature drop,

TEMP = temporary value,

TEMPDM = moment due to a temperature drop,

TEMPDN = thrust due to a temperature drop,

TEMPDV = shear due to a temperature drop,

TEMPRM = moment due to a temperature rise,

TEMPRN = thrust due to a temperature rise,

TEMPRV = shear due to a temperature rise,

TF = temporary flange thickness,

TFLG = flange thickness,

TFMAX = maximum flange thickness,

TFMIN = minimum flange thickness,

THED = arch slope angles at ends of small segments of arch rib,

THETA = arch slope angles at design points,

THRUST = thrust influence lines,

TLOAD = type of live load for preliminary designs,

TMAXM = maximum moment due to a temperature change,

TMAXN = maximum thrust due to a temperature change,

TMAXV = maximum shear due to a temperature change,

TMINM = minimum moment due to a temperature change,

TMINN = minimum thrust due to a temperature change,

TOLCOST = objective function cost tolerance,

TOLD = web depth tolerance,

TOLDEFL = tolerable live load deflection,

TOLH = arch height tolerance,

TR = temperature rise,

TW = temporary web thickness,

TWEB = web thickness,

TWMIN = minimum web thickness,

TYPE = temporary arch type,

V = temporary shear,

V1 = temporary maximum live load shear,

V2 = temporary minimum live load shear,

VD = shear due to a temperature drop,

VERT = vertical reaction component,

VERTS = vertical reaction component influence line,

VLEFT = vertical force at left support,

VOLUME = volume of flanges and webs,

VR = shear due to a temperature rise,

VRIGHT = vertical force at right support,

W = temporary uniform lane load, temporary flange width,

WC = weight of column per foot,

WD = deck weight per foot,

WDECK = weight of deck per foot,

WEIGHT = total weight of rib plus stiffeners,

WGTR = average weight per foot of rib plus stiffeners,

WI = uniform lane load plus impact,

WIDTH = flange width,

WINDM = moment due to wind on structure,

WINDN = thrust due to wind on structure,

WINDV = shear due to wind on structure,

WL = uniform lane load,

WLM = moment due to wind on fictitious live load,

WLN = thrust due to wind on fictitious live load,

WLV = shear due to wind on fictitious live load,

WR = temporary total weight of rib plus stiffeners,

WS = sidewalk live load,

X = horizontal distance to design points,

X1, X2 = temporary distances for computing the area under an influence line, horizontal distance to columns L1 and L2,

XC = horizontal distance to column point,

XD = horizontal distance to ends of small segments of arch rib,

XT = temporary X value in geometry program,

XXC = temporary distance to column points,

Y = vertical distance to design points, temporary magnitude of function for Simpson integration,

YC = vertical distance to column point,

YD = vertical distance to ends of small segments of arch rib,

YT = temporary Y value in geometry program.

APPENDIX E

COMPUTER PROGRAM USER'S GUIDE

General Discussion

The total program is made up of 22 subprograms which were checked individually before they were combined to make up the main program. The main control program can be divided into six parts which are (1) data input, (2) initialization of variables, (3) analysis and design computations, (4) search procedure, (5) final design computations and (6) data output. The same six divisions are used for the flow diagram which is discussed in this appendix. Parts 1, 5 and 6 are used only once during an arch design; however, parts 2, 3 and 4 are used several times.

The computer program listed in Appendix F is written in ALGOL for the Burroughs B-5500 computer facilities at Georgia Institute of Technology. This program uses the method of conjugate directions for a search of the objective function surface assuming stress rather than live load deflection governs the design surface and the ravine search (called RIDGE) is used in case live load deflection is excessive. A computer translation was made from ALGOL to FORTRAN.

The computer time required to produce a good design close to the optimum varies between 10 to 15 minutes on the Burroughs B-5500. Approximately two minutes are required to compile the program. The time required to make the computations of a single arch design involving steps 2 and 3 as given previously is estimated to be three seconds on the Burroughs B-5500.

All of the variables used in the following discussion are variables that are used in the computer program. In most cases, descriptive names or names similar to those of the thesis text are used.

Subprograms

The 22 subprograms are listed and discussed in the order in which they appear in the main program.

PROCEDURE GEOMETRY. This subprogram computes all values of the geometric configuration of the arch that are required. Two types of arches are considered for the present computer program. They are the parabolic arch (TARCH = 1) and the circular arch (TARCH = 3).

PROCEDURE AREAP. This subprogram computes the AREA (sq inches) of the rib cross section at all design points.

PROCEDURE INERTIAP. This subprogram computes the moment of INERTIA ($in.^4$) of the rib cross section at all design points.

PROCEDURE STRESSP. This subprogram computes the stress (ksi) produced by THRUST (kips) and MOM (kip ft).

PROCEDURE SIMPSON. This subprogram makes numerical integrations of several quantities throughout the program.

PROCEDURE SUPPORTFORCES. This subprogram computes the magnitudes of the influence lines for the reaction components (HORIZS and VERTS) of the left, high support.

PROCEDURE DEFLECTIONS. This subprogram uses the influence lines for the reaction components in order to compute the magnitudes of the deflection (DEFL, ft/kip) influence lines at each of the column points.

PROCEDURE INFLUENCELINES. This subprogram uses the influence lines for the reaction components in order to compute the magnitudes of the MOM, THRUST, SHEAR and STRESS influence lines. The MOM, THRUST, and SHEAR influence lines have no units; however, the units of the STRESS influence lines are ksi when the loads are in kips.

PROCEDURE INTERSTATE. This subprogram computes the moment (LMAXM, LMINM) and thrust (LMAXN, LMINN) for the maximum and minimum stress condition (LMAXS, LMINS) produced by interstate live load including impact. In addition, the subprogram computes the maximum and minimum deflection (LMAXD, LMIND), shear (LMAXV, LMINV) and the maximum column force (FCLL) including impact.

PROCEDURE TRUCK. This subprogram computes the magnitudes of the values produced by truck loading including impact and compares them with those produced by interstate loading so that the appropriate value can be selected.

PROCEDURE LANE. This subprogram computes the magnitudes of the values produced by lane loading including impact and compares them with those produced by the interstate and truck loading so that the appropriate value can be selected. If the sidewalk live loading (WS) is not zero, then the moment (MAXSWM, MINSWM), thrust (MAXSWN, MINSWN) and shear (MAXSWV, MINSWV) are computed at each of the design points; and the maximum live load column force (FCSW) produced by the sidewalk loading is computed at each column point. These sidewalk values do not include impact, and they are added to the governing values from interstate, truck or lane loading.

PROCEDURE RIBDL. This subprogram computes the moment (DLM), thrust (DLN), shear (DLV) and deflection (DLD) produced by the rib weight.

PROCEDURE COLUMNDL. This subprogram is supplied with (1) the column force (FC) resulting from deck dead load, deck bracing and arch rib bracing and (2) the assumed weight per foot of the column plus column bracing (WC). These two loads are used to compute moment, thrust, shear and deflection which are added to the results from the subprogram RIBDL.

PROCEDURE LF. This subprogram computes the moment (LFM), thrust (LFN) and shear (LFV) resulting from a longitudinal force (LONGF) which is placed at an elevation HLF above the left, high support or may be carried to the arch rib through two braced columns at column points L1 and L2.

PROCEDURE TEMPERATURE. This subprogram is supplied with a temperature rise (TR) and a temperature drop (TD) in degrees Fahrenheit. These two temperature values are used to compute moment (MR, MD), thrust (NR, ND) and shear (VR, VD) resulting from a temperature change.

PROCEDURE WIND. This subprogram computes the moment (WINDM), thrust (WINDN) and shear (WINDV) due to wind on the structure; and it computes the moment (WLM), thrust (WLN) and shear (WLV) due to wind on a fictitious live load.

PROCEDURE EARTHQUAKE. This subprogram computes the moment

(EQM), thrust (EQN) and shear (EQS) due to an earthquake loading.

PROCEDURE SHEARP. This subprogram combines the shear forces resulting from all loads and computes the governing shear force (CRITV) at each design point.

PROCEDURE ALSTRESS. This subprogram computes the allowable stress (ALLOWABLE) at each design point.

PROCEDURE CRITICALSTR. This subprogram combines the stresses due to all loads in order to arrive at the governing stress (CRITSTR), governing moment (CRITM) and governing thrust (CRITN). In addition, this subprogram controls the search for the allowable stress.

PROCEDURE SECTIONADJUST. This subprogram makes adjustments in the flange thickness (TFLG) of the arch rib so that the resulting governing stress is equal to the allowable stress at all design points.

PROCEDURE COSTP. This subprogram computes the rib weight (RIBWGT) and uses the estimated weight of the stiffening elements (STIFFENERS) in order to compute the total cost (COST) of the arch rib.

Required Input Data

The required input data must be placed in a free field format on the cards. This means that the data may be placed anywhere on the card or cards so long as each piece of data is given and is followed by a comma. For explanation and checking purposes, the data have been divided into 10 groups or, as is usually the case, 10 cards. The last five groups may require several cards to each group depending upon how many design points and column points are used. All computer input is read into the computer storage at the very beginning of the program.

A list of the 10 groups of input appears as follows:

1. NC, NP, TARCH, L1, L2,
2. SPAN, HEIGHT, DELH, HDECK, HCG, HLF, DECKD, HORIZD, DEPTH, WIDTH,
3. PI, PT1, PT2, PL, WL, WS, WDECK, LONGF, TR, TD, RIDWGT,
4. FY, FU, ETA, CEQ, PRICE1, PRICE2,
5. ALFAK, TOLH, TOLD, TOLDEFL, TOLCOST,
6. XC (0, 1, 2, 3, ... NC),
7. X (0, 1, 2, 3, ... NP),
8. TFLG (0, 1, 2, 3, ... NP),
9. FC (1, 2, 3, 4, ... NC-1),
10. WC (1, 2, 3, 4, ... NC-1).

The 10 groups of input are discussed as follows:

Group 1

NC = number of columns (0, 1, 2, 3, ...),

NP = number of design points (0, 1, 2, 3, ...),

All column points must be design points.

TARCH = type of arch,

TARCH = 1, parabolic arch,

TARCH = 3, circular arch,

TLOAD = type of live load to be used for preliminary designs,

TLOAD = 1, INTERSTATE,

TLOAD = 2, TRUCK,

TLOAD = 3, LANE,

TLOAD = 4, INTERSTATE AND TRUCK,

TLOAD = 5, TRUCK AND LANE,

TLOAD = 6, INTERSTATE, TRUCK AND LANE,

L1 and L2 = columns at which LONGF is carried to the arch rib. These two points may be zero if LONGF = 0.

<u>Group 2</u>

SPAN = horizontal distance in feet between arch supports,

HEIGHT = vertical distance in feet from the left, high support to the crown on the arch rib,

DELH = positive magnitude of the vertical distance in feet from the high support to the low support for an unsymmetrical arch,

HDECK = height in feet to the deck surface from the left, high support,

HCG = height in feet to the center of mass of the bridge deck including the weight of the concrete deck and longitudinal stringers. This value may be zero if EARTHQUAKE forces are not to be computed.

HLF = height in feet to the LONGF assuming the LONGF is carried to the arch rib through the deck. HLF will usually be approximately equal to HDECK.

DECKD = total deck depth in feet from the bottom of the longitudinal stringers to the top of the parapet. This value is used for wind loading on deck.

HORIZD = horizontal distance in feet between the two arch ribs,

DEPTH = rib depth in inches,

WIDTH = rib flange width inches.

Group 3

PI = portion of the interstate live load in kips that is carried to the arch rib;

PI = 24 kips x live load distribution factor.

PT1 and PT2 = portion of the truck live load in kips that is carried to the arch rib,

PL = portion of the concentrated lane live load in kips for moment computations that is carried to the arch rib,

WL = portion of the uniform lane live load in kips per foot that is carried to the arch rib,

WS = portion of the uniform sidewalk live load in kips per foot that is carried to the arch rib,

WDECK = total weight of the bridge deck including the longitudinal stringers in kips per foot for earthquake computations,

LONGF = magnitude of the longitudinal force in kips from the tractive force of the live load,

TR = assumed temperature rise (^{o}F),

TD = assumed temperature drop (^{o}F),

RIBWGT = estimated rib weight in kips for the first preliminary design,

Group 4

FY = yield strength of the steel in ksi,

FU = ultimate strength of the steel in ksi,

ETA = factor of safety based on the yield point,

CEQ = if EARTHQUAKE forces are to be considered then CEQ, according to AASHO Specifications [64], equals the following:

CEQ = 0.02 for structures founded on spread footings on material rated as 4 tons or more per square foot,

CEQ = 0.04 for structures founded on spread footings on material rated as less than 4 tons per square foot,

CEQ = 0.06 for structures founded on piles.

PRICE1 = price of steel per pound for the rib flanges and webs,

PRICE2 = price of steel per pound for the stiffeners and internal rib diaphragms,

Group 5

ALFAK = initial basic step size for the stepping search,

TOLH = tolerance for the variation of HEIGHT for stopping the search,

TOLD = tolerance for the variation of DEPTH for stopping the search,

TOLDEFL = tolerable deflection due to live load plus impact (in./in. = 1/800 of the span except in urban areas where the bridge is used in part by pedestrians whereon the ratio shall preferably be 1/1000),

TOLCOST = tolerance for the variation of the total COST of the rib at convergence.

Group 6

XC = horizontal distance in feet from the left, high support to each of the columns.

Group 7

X = horizontal distance in feet from the left, high support to each of the design points.

Group 8

TFLG = estimated thickness of the flange in inches for the initial design.

Group 9

FC = force at the column point in kips due to the dead weight of the bridge deck including the concrete deck, longitudinal stringers, deck bracing and arch rib bracing.

Group 10

WC = assumed weight per foot of column plus column bracing (kips/ft).

Output Data

Although all data required to complete the arch rib and bridge design are printed, some designers would prefer more which can be obtained easily with a slight amount of additional programming. All output is printed at the very end of the program. The output is grouped into 22 groups, and a list of these groups appears as follows:

1. I, CI(I), DI(I), HI(I), DEFLI(I), RI(I), SI(I), (I = 1, 2, 3, ...DESIGNS),
2. I, C(I), D(I), H(I), (I = 1, 2, 3, ... DESIGN),
3. NC, NP, ND, TARCH, L1, L2,

   SPAN, SPANS, LGTH, HEIGHT, DELH, HDECK, HCG, HLF, DECKD,

   HORIZD,
4. DEPTH WIDTH, TWEB,
5. PI, PT1, PT2, PL, WL, WS, WDECK, LONGF, TR, TD, CEQ,

6. FY, FU, ETA,

   PRICE1, PRICE2,

   ALFAK, TOLH, TOLD, TOLDEFL, TOLCOST,

7. HOPT, DOPT,

   RIBWGT, STIFFENERS, COST,

8. XC (0, 1, 2, 3, ...NC),

9. X (0, 1, 2, 3, ...NP),

10. Y (0, 1, 2, 3, ...NP),

11. THETA (0, 1, 2, 3, ...NP),

12. TFLG (0, 1, 2, 3, ...NP),

13. CRITSTR (0, 1, 2, 3, ...NP),

14. ALLOWABLE (0, 1, 2, 3, ...NP),

15. CRITM (0, 1, 2, 3, ...NP),

16. CRITN (0, 1, 2, 3, ...NP),

17. CRITV (0, 1, 2, 3, ...NP),

18. FC (1, 2, 3, 4, ...NC-1),

19. FCLL (1, 2, 3, 4, ...NC-1),

20. DLD (0, 1, 2, 3, ...NC),

21. LMAXD (0, 1,2,3, ...NC),

22. LMIND (0, 1, 2, 3, ...NC).

Those portions of output that were not previously explained for the input items or those whose identity has changed are explained below.

Group 1--All designs for search steps.

I = number of the design,

CI = COST of arch rib (dollars),

DI = DEPTH of rib section (in.),

HI = HEIGHT of arch rib (ft),

DEFLI = magnitude of the largest upward or downward live load deflection (ft). This value will be zero until the search is completed assuming stress controls the design.

RI = RIBWGT which includes the weight of the flanges and webs (kips),

SI = approximate STIFFENER weight which includes the weight of the diaphragms and longitudinal stiffeners (kips),

Group 2 --These values have been called "improved search steps;" however, they may not be an improvement over previous values at all. This is entirely due to "noise." When the direction of the search is changed, the magnitude of the function is recomputed; and often the new value exceeds the previous low. Noise is still present in most of the designs; however, special attention is given to the final design in order to reduce noise to a small amount.

I = number of the design,

C = COST of arch rib (dollars),

D = DEPTH of rib section (in.),

H = HEIGHT of arch rib (ft).

Group 3-- Arch shape properties

ND = number of segments that the arch rib is divided into for integration purposes

(ND = 10 x NP),

SPANS = twice the horizontal distance in feet from the left, high support to the crown of the arch rib,

LGTH = total length of the arch rib in feet.

Group 4--Arch rib properties

TWEB = web thickness in inches.

Group 7--Resulting final design values of rib dimensions, rib weight and cost estimate

HOPT = final HEIGHT,

DOPT = final DEPTH,

RIBWGT = final rib weight in pounds,

STIFFENERS = the final estimated weight of the longitudinal stiffeners and diaphragms in pounds,

COST = the final estimated cost of the arch rib.

Group 11

THETA = the slope angle of the arch rib at each design point in radians.

Group 12

TFLG = the resulting flange thickness in inches for the final design.

Group 13

CRITSTR = the resulting governing stress at each design point in ksi.

Group 14

ALLOWABLE = the computed allowable stress at each design point in ksi.

Group 15

CRITM = the resulting moment for the final design at each design point in kip ft.

Group 16

CRITN = the resulting thrust for the final design at each design point in kips.

Group 17

CRITV = the resulting governing shear for the final design at each design point in kips.

Group 19

FCLL = the resulting column force in kips due to live load including impact plus sidewalk live load without impact.

Group 20

DLD = the dead load deflection in feet at each column point due to the total dead weight of the arch bridge.

Group 21

LMAXD = the maximum downward deflection in feet due to live load plus impact, not including sidewalk live load.

Group 22

LMIND = the maximum upperward deflection in feet due to live load plus impact, not including sidewalk live load.

Flow Diagram

All subprograms precede the main control program and these subprograms are used by the control program in the manner shown in the flow diagram. The six parts of the control program can be briefly explained as follows:

Part 1. All data is read into computer storage.

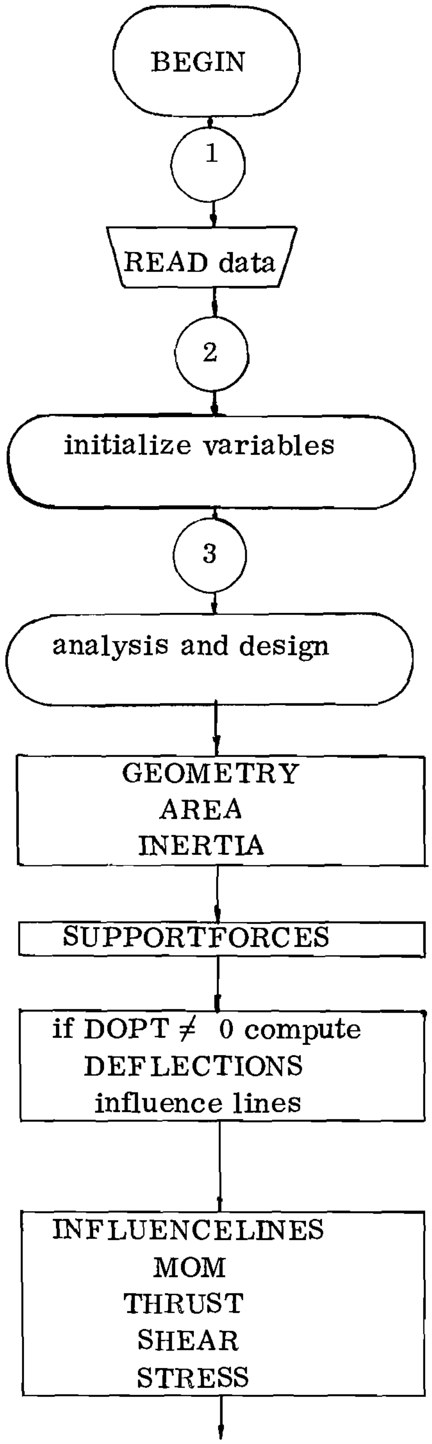
Part 2. Most of the variables that require initialization are set to their initial values in part 2. Variables that require initialization are as follows: control values, maximum and minimum values, some input values plus several variables that require being set to zero. Many of the variables require initialization for each cycle of analysis redesign.

Part 3. This part of the computer program includes the major portion of all computations. All required geometric computations, loads, stresses and flange thickness adjustments are made for each cycle of analysis redesign.

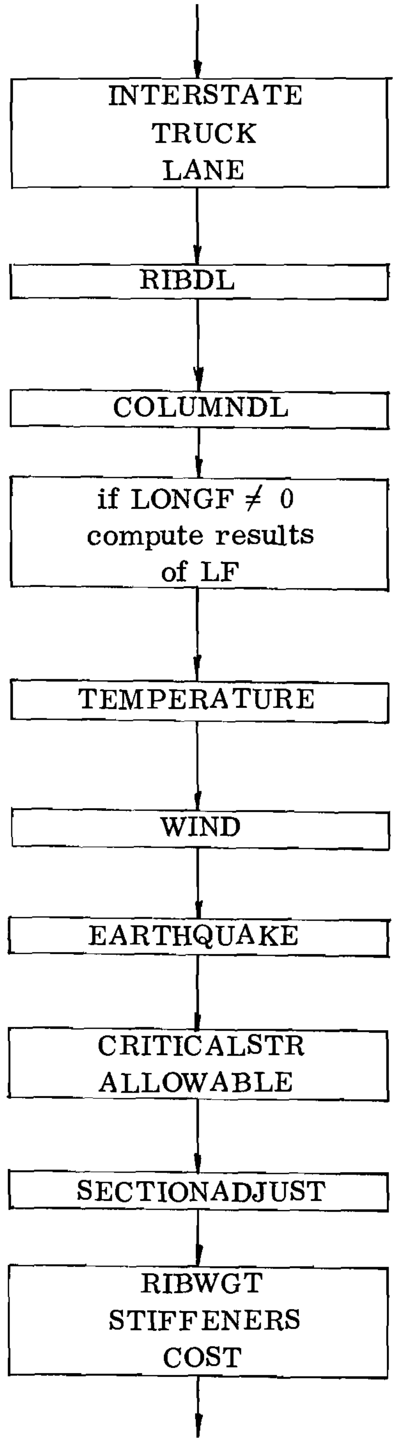
Part 4. The arch height and web depth are adjusted by searching along the objective function surface seeking the least cost design.

Part 5. When the tolerance values are satisfied the final design is made.

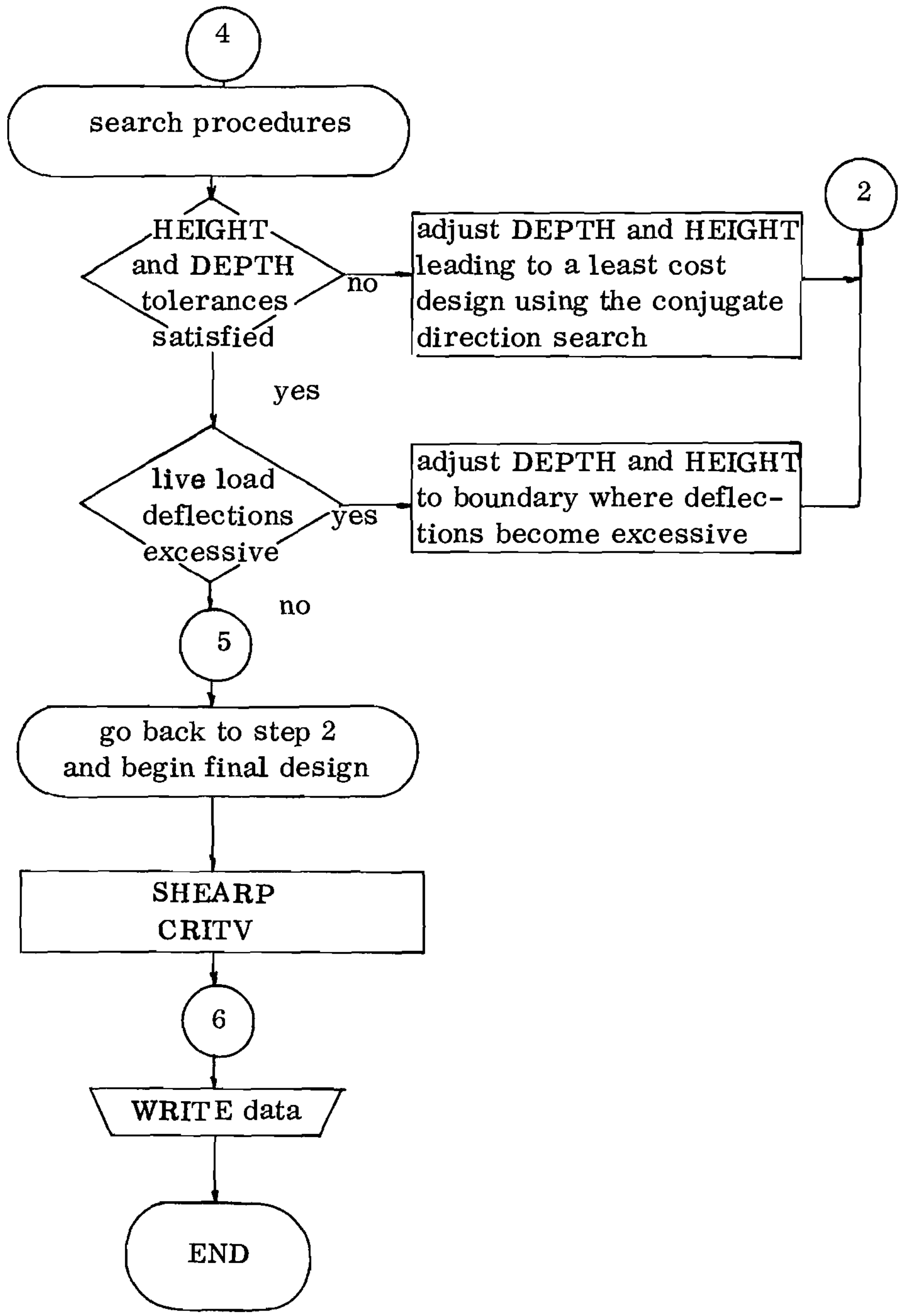
Part 6. All data required to complete the arch rib and bridge are printed here.



Flow Diagram for Computer Program



Flow Diagram for Computer Program (Continued)


4
search procedures
HEIGHT and DEPTH tolerances satisfied
no
adjust DEPTH and HEIGHT leading to a least cost design using the conjugate direction search
2
yes
live load deflections excessive
yes
adjust DEPTH and HEIGHT to boundary where deflections become excessive
no
5
go back to step 2 and begin final design
SHEARP
CRITV
6
WRITE data
END

APPENDIX F

COMPUTER PROGRAM

```
BEGIN
%  CONTROL PROGRAM FOR A TWO HINGED ARCH DESIGN
   FILE IN CF (2,10);  FILE OUT PF 15(2,15);
   INTEGER DES,DESIGN,DESIGNS,DESN,GOMIN,GOSEARCH,I,J,L,L1,L2,LC,LR,
      NC,ND,NP,NG,NO,NEG,TARCH,TLOAD,NRIDGE,DES5,LCC,NO1,NC1,NP1;
   REAL SPAN,HEIGHT,DELH,HDECK,HCG,HLF,DECKD,HORIZD,DEPTH,WIDTH,
      PI,PT1,PT2,PL,WL,WS,WDECK,LONGF,TR,TD,RIBWGT,
      FY,FU,ETA,CEQ,PRICE1,PRICE2,
      ALFAK,TOLH,TOLD,TOLDEFL,TOLCOST,
      SPANS,SPANL,HOPT,RADIUS,LGTH,DOPT,TWEB,RATIO,TF,TFMIN,TWMIN,TFMAX,
      GD,GH,GK,STEPK,IDEFL,LDEFL,COST,WR,STIFFENERS,
      FYY,FYOPT,STR,SUM1,TEMP;
   REAL ARRAY XC,WC,FC[0:15],X,Y,TFLG[0:15],
      AREA,INERTIA,DELS,THETA,HORIZS,VERTS[0:15],
      XD,YD,THED,SINE,COSE,SUM,MI,MJ,MM,NI,NJ,NN[0:150],
      MOM,THRUST,SHEAR,STRESS,DEFL[0:15,0:12],
      C,D,H,DEFLECTION,R,S[0:100],CI,DI,HI,DEFLI,RI,SI[0:200],
      LMAXD,LMIND,FCLL,FCSW,FC1,FFC,DLD,XXC,YC,DELC[0:15],
      LENGTH,DIST,CRITSTR,CRITM,CRITN,CRITV,ALLOWABLE,CYCLES,LMAXS,
      LMINS,LMAXM,LMINM,LMAXN,LMINN,LMAXV,LMINV,MAXSWM,MINSWM,MAXSWN,
      MINSWN,MAXSWV,MINSWV,S1,S2,M1,M2,N1,N2,V1,V2,D1,D2,DLM,DLN,DLV,
      LFM,LFN,LFV,WINDM,WINDN,WINDV,WLM,WLN,WLV,EQM,EQN,EQS,
      TEMPRM,TEMPRN,TEMPRV,TEMPDM,TEMPDN,TEMPDV,MAXV,TMAXV,MAXM,MAXN,
      MINM,MINN,MAXS,MINS,DELSTR,ASTR,TMAXM,TMAXN,TMINM,TMINN[0:15];
   LABEL START,FINISH,CONV,CONJ,RIDGE,LAB1,LAB2,
      CL1,CL2,CL3,CL4,
      RL1,RL2,RL3,RL4,RL5,RL7,RL8,RL9,
      RD1,RD2,RD3;
   SWITCH SEARCH←CONJ,RIDGE,CONV;
   SWITCH CSEARCH←CL1,CL2,CL4;
   SWITCH RSEARCH←RL1,RL3,RL4,RL7,RL8,RL9;
   FORMAT FMT1(//X10"ARCH SHAPE PROPERTIES"/),
      FMT2(//X10"ARCH RIB PROPERTIES"/),
      FMT3(//X10"ARCH LOADS"/),
      FMT4(//X10"STRENGTH, PRICE, AND TOLERANCE VALUES"/),
      FMT5(//X10"OPTIMUM VALUES-RIB DIM., RIB WGT., COST ESTIMATE"/),
      FMT6(//X10"XC COORDINATES"/),
      FMT7(//X10"X COORDINATES"/),
      FMT8(//X10"Y COORDINATES"/),
      FMT9(//X10"RIB SLOPE ANGLES AT DESIGN POINTS"/),
      FMT10(//X10"FLANGE THICKNESS"/),
      FMT11(//X10"CRITICAL STRESS"/),
      FMT12(//X10"CRITICAL MOMENT"/),
      FMT13(//X10"CRITICAL THRUST"/),
      FMT14(//X10"CRITICAL SHEAR"/),
      FMT15(//X10"COLUMN FORCE DUE TO DEAD LOAD"/),
      FMT16(//X10"COLUMN FORCE DUE TO LIVE LOAD WITH IMPACT PLUS "
         "SIDEWALK LIVE LOAD"/),
      FMT17(//X10"DEFLECTIONS DUE TO DEAD LOAD"/),
      FMT18(//X10"DEFLECTIONS DUE TO LIVE LOAD"/),
      FMT19(//X10"**** TWO HINGED HIGHWAY ARCH DESIGN ****"//),
      FMT20(//X10"ALLOWABLE STRESS"/),
      FMT21(//X10"ALL SEARCH STEPS"//X9"NUMBER, COST, DEPTH, HEIGHT, "
         "LIVE LOAD DEFLECTION, RIB WEIGHT AND STIFFENER WEIGHT"//),
      FMT22(//X10"IMPROVED SEARCH STEPS"//X9"NUMBER, COST, DEPTH, "
         "HEIGHT"//),
      FMT30(10I10/),
      FMT31(12F9.4/),
      FMT32(12F9.3/),
      FMT33(12F9.2/),
      FMT34(12F9.1/),
      FMT35(12F9.0/),
```

```
        FMT36(I10,F12.2,3F10.4,2F12.2);
PROCEDURE GEOMETRY(NP,ND,TYPE,SPAN,SPANS,HEIGHT,DELH,X,Y,THETA,DELS,
    XD,YD,THED,LENGTH);
%   TYPE 1 FOR A SYMMETRICAL PARABOLIC ARCH
%   TYPE 2 FOR AN UNSYMMETRICAL PARABOLIC ARCH
%   TYPE 3 FOR A SYMMETRICAL CIRCULAR ARCH
%   TYPE 4 FOR AN UNSYMMETRICAL CIRCULAR ARCH
    VALUE NP,TYPE,SPAN,DELH;
    INTEGER NP,ND,TYPE;
    REAL SPAN,SPANS,HEIGHT,DELH;
    REAL ARRAY X,Y,THETA,DELS,LENGTH[0],    XD,YD,THED[0];
    BEGIN
        INTEGER I,J,N,N1,N2;
        REAL K,SPANL,RADIUS,LENGTH1,F,F1,SQ,XT,YT,EPS;
        LABEL LAB1,LAB2,LAB3,LAB4,LAB5;
        ND←10×NP;
        N←NP-1;
        IF TYPE=1 OR TYPE=2 THEN
        BEGIN
            IF DELH=0 THEN SPANL←SPAN ELSE
            SPANL←2×SPAN/(SQRT(HEIGHT/(HEIGHT+DELH))+1);
            SPANS←SPAN+SPAN-SPANL;
            K←4×HEIGHT/(SPANS×SPANS);
            Y[0]←0;
            Y[NP]←-DELH;
            FOR I←1 STEP 1 UNTIL N DO
                Y[I]←4×HEIGHT/SPANS×(X[I]-X[I]×X[I]/SPANS);
            FOR I←0 STEP 1 UNTIL NP DO
                THETA[I]←ARCTAN(2×K×(SPANS/2-X[I]));
            FOR I←0 STEP 1 UNTIL NP DO
            BEGIN
                XT←ABS(SPANS/2-X[I]);
                YT←HEIGHT-Y[I];
                IF YT=0 THEN GO TO LAB3;
                SQ←SQRT(4×XT×XT+YT×YT);
                LENGTH[I]←SQ/2+YT×YT/(4×XT)×LN((2×XT+SQ)/YT);
    LAB3:END;
            FOR I←0 STEP 1 UNTIL NP DO
                IF X[I]<SPANS/2 THEN LENGTH[I]←-LENGTH[I] ELSE GO TO LAB1;
    LAB1:FOR I←0 STEP 1 UNTIL N DO
                DELS[I]←ABS(LENGTH[I]-LENGTH[I+1])/10;
            FOR I←0 STEP 1 UNTIL N DO
            BEGIN
                XT←ABS(SPANS/2-X[I]);
                LENGTH1←LENGTH[I];
                N1←10×I;
                N2←10×(I+1);
                FOR J←N1 STEP 1 UNTIL N2 DO
                BEGIN
             LAB2:SQ←SQRT(4+K×K×XT×XT);
                    IF ABS(XT)≤0.001 THEN GO TO LAB5;
                    F←XT×SQ/2+(K×K×(XT*3)/4)×LN((2+SQ)/(K×XT))-ABS(LENGTH1);
                    F1←((K×XT)*2)/2;
                    F1←SQ/2+F1/SQ+F1×F1/((2+SQ)×SQ)-F1/2
                        +1.5×F1×LN((2+SQ)/(K×XT));
                    EPS←F/F1;
                    XT←XT-EPS;
                    IF ABS(EPS)>0.001 THEN GO TO LAB2;
             LAB5:YD[J]←HEIGHT-K×XT×XT;
                    IF LENGTH1<0 THEN XD[J]←SPANS/2-XT ELSE XD[J]←SPANS/2+XT;
                    LENGTH1←LENGTH1+DELS[I];
                    XT←ABS(SPANS/2-XD[J]-DELS[I])
```

```
            END
         END;
         FOR J←0 STEP 1 UNTIL ND DO
            THED[J]←ARCTAN(2×K×(SPANS/2-XD[J]))
      END;
LAB4:IF TYPE=3 OR TYPE=4 THEN
      BEGIN
         IF DELH=0 THEN SPANL←SPAN ELSE
            SPANL←(SPAN-SQRT((SPAN×SPAN×HEIGHT+HEIGHT×DELH×DELH)/
               (HEIGHT+DELH)))×2×(HEIGHT+DELH)/DELH;
         SPANS←SPAN+SPAN-SPANL;
         THETA[0]←2×ARCTAN(2×HEIGHT/SPANS);
         RADIUS←SPANS/2/SIN(THETA[0]);
         IF HEIGHT+DELH>RADIUS THEN
         BEGIN
            HEIGHT←RADIUS-DELH;
            GO TO LAB4
         END;
         Y[0]←0;
         Y[NP]←-DELH;
         FOR I←1 STEP 1 UNTIL NP DO
            THETA[I]←ARCTAN((SPANS/2-X[I])/
               SQRT(RADIUS*2-(SPANS/2-X[I])*2));
         FOR I←1 STEP 1 UNTIL N DO
            Y[I]←HEIGHT-(SPANS/2-X[I])×SIN(THETA[I]/2)/COS(THETA[I]/2);
         FOR I←0 STEP 1 UNTIL NP DO LENGTH[I]←-THETA[I]×RADIUS;
         FOR I←0 STEP 1 UNTIL N DO
            DELS[I]←ABS(LENGTH[I]-LENGTH[I+1])/10;
         FOR I←0 STEP 1 UNTIL N DO
         BEGIN
            LENGTH1←LENGTH[I];
            N1←10×I;
            N2←10×(I+1);
            FOR J←N1 STEP 1 UNTIL N2 DO
            BEGIN
               THED[J]←-LENGTH1/RADIUS;
               XD[J]←SPANS/2-RADIUS×SIN(THED[J]);
               YD[J]←HEIGHT-RADIUS×(1-COS(THED[J]));
               LENGTH1←LENGTH1+DELS[I]
            END
         END
      END;
   END;
PROCEDURE AREAP(DEPTH,WIDTH,TWEB,TFLG,AREA);
   VALUE DEPTH,WIDTH,TWEB,TFLG;
   REAL DEPTH,WIDTH,TWEB,TFLG,AREA;
   BEGIN
      AREA←2×(WIDTH×TFLG+DEPTH×TWEB)
   END;
PROCEDURE INERTIAP(DEPTH,WIDTH,TWEB,TFLG,INERTIA);
   VALUE DEPTH,WIDTH,TWEB,TFLG;
   REAL DEPTH,WIDTH,TWEB,TFLG,INERTIA;
   BEGIN
      INERTIA←2×(TWEB×(DEPTH*3)/12+WIDTH×TFLG×(((DEPTH+TFLG)/2)*2))
      +WIDTH×TFLG×TFLG×TFLG/6
   END;
PROCEDURE STRESSP(THRUST,MOM,AREA,INERTIA,DEPTH,TFLG,STR);
%  THIS PROCEDURE COMPUTES STRESS DUE TO BENDING AND THRUST
   VALUE MOM,THRUST,AREA,INERTIA,DEPTH,TFLG;
   REAL MOM,THRUST,AREA,INERTIA,DEPTH,TFLG,STR;
   BEGIN
      STR←THRUST/AREA+MOM×12×(DEPTH/2+TFLG)/INERTIA
```

```
   END;
PROCEDURE SIMPSON(N1,N2,H,Y,SUM);
%  PROCEDURE SIMPSON COMPUTES "SUM=AREA UNDER THE CURVE" FOR CASES
%  WHERE THE INCREMENT "H" IS CONSTANT, THE CURVE IS SINGLE VALUED AT
%  ALL POINTS AND "N2-N1" IS EVEN
   VALUE H;
   INTEGER N1,N2;
   REAL H,SUM;
   REAL ARRAY Y[0];
   BEGIN
      INTEGER I;
      REAL TEMP;
      TEMP←Y[N1]+Y[N2];
      N1←N1+1;
      N2←N2-1;
      FOR I←N1 STEP 2 UNTIL N2 DO
         TEMP←TEMP+4×Y[I];
      N1←N1+1;
      N2←N2-1;
      FOR I←N1 STEP 2 UNTIL N2 DO
         TEMP←TEMP+2×Y[I];
      SUM←H×TEMP/3
   END;
PROCEDURE SUPPORTFORCES(NC,NP,SPAN,HEIGHT,DELH,SUM1,AREA,INERTIA,XC,X,
   XD,Y,YD,DELS,HORIZS,VERTS,SUM);
%  COMPUTES SUPPORT FORCES AT COL PTS FOR ARCHES WITH HINGED SUPPORTS
   VALUE NC,NP,SPAN,HEIGHT,DELH;
   INTEGER NC,NP;
   REAL SPAN,HEIGHT,DELH,SUM1;
   REAL ARRAY AREA,INERTIA,XC,X,Y,DELS,HORIZS,VERTS[0],
      XD,YD,SUM[0];
   BEGIN
      INTEGER I,J,JJ,N,ND,N1,N2;
      REAL      SUM2,SUM3,ITEMP,ATEMP,VLEFT,VRIGHT,SUMTEMP,H;
      LABEL LAB1,LAB2;
      ND←NP×10;
      N←NP-1;
      SUM1←0;
      FOR I←0 STEP 1 UNTIL ND DO
         SUM[I]←(YD[I]+DELH×XD[I]/SPAN)*2;
      FOR I←0 STEP 1 UNTIL N DO
      BEGIN
         IF INERTIA[I]<INERTIA[I+1] THEN ITEMP←INERTIA[I+1]
            ELSE ITEMP←INERTIA[I];
         H←DELS[I]/ITEMP;
         N1←10×I;
         N2←10×(I+1);
         SIMPSON(N1,N2,H,SUM,SUMTEMP);
         SUM1←SUM1+SUMTEMP
      END;
      SUM1←SUM1×144;
      SUM2←0;
      IF HEIGHT/SPAN<0.15 THEN
         FOR I←0 STEP 1 UNTIL N DO
         BEGIN
            IF AREA[I]<AREA[I+1] THEN ATEMP←AREA[I+1]
               ELSE ATEMP←AREA[I];
            SUM2←SUM2+10×DELS[I]/ATEMP
         END;
      SUM2←SUM2×12;
      FOR I←1 STEP 1 UNTIL NC1  DO
      BEGIN
```

```
            VLEFT←(SPAN-XC[I])/SPAN;
            VRIGHT←XC[I]/SPAN;
            SUM3←0;
            FOR J←0 STEP 1 UNTIL N DO
            BEGIN
               IF INERTIA[J]<INERTIA[J+1] THEN ITEMP←INERTIA[J+1]
                  ELSE ITEMP←INERTIA[J];
               H←DELS[J]/ITEMP;
               N1←10×J;
               N2←10×(J+1);
               IF XC[I]≤X[J] THEN GO TO LAB1;
               FOR JJ←N1 STEP 1 UNTIL N2 DO
                  SUM[JJ]←VLEFT×XD[JJ]×(YD[JJ]+DELH×XD[JJ]/SPAN);
               GO TO LAB2;
         LAB1:FOR JJ←N1 STEP 1 UNTIL N2 DO
                  SUM[JJ]←VRIGHT×(SPAN-XD[JJ])×(YD[JJ]+DELH×XD[JJ]/SPAN);
         LAB2:SIMPSON(N1,N2,H,SUM,SUMTEMP);
               SUM3←SUM3+SUMTEMP
            END;
            SUM3←SUM3×144;
            HORIZS[I]←SUM3/(SUM1+SUM2);
            VERTS[I]←(SPAN-XC[I])/SPAN-HORIZS[I]×DELH/SPAN
         END;
         HORIZS[0]←HORIZS[NC]←VERTS[NC]←0;
         VERTS[0]←1
      END;
   PROCEDURE DEFLECTIONS(NC,NP,ND,XC,HORIZS,VERTS,AREA,INERTIA,
      XD,YD,THED,DELS,DEFL,MI,MJ,MM,NI,NJ,NN,COSE,SINE);
   % COMPUTES DEFLECTIONS AT COLUMN POINTS DUE TO A UNIT LOAD
      VALUE NC,NP,ND;
      INTEGER NC,NP,ND;
      REAL ARRAY XC,HORIZS,VERTS[0],
         AREA,INERTIA,DELS[0],    DEFL[0,0],
         XD,YD,THED,MI,MJ,MM,NI,NJ,NN,COSE,SINE[0];
      BEGIN
         INTEGER I,J,K,N,N1,N2;
         REAL A,DEFM,DEFN,H,IR,TEMP;
         N←NC-1;
         FOR I←0 STEP 1 UNTIL NP DO
         FOR J←0 STEP 1 UNTIL NC DO DEFL[I,J]←0;
         FOR J←0 STEP 1 UNTIL ND DO
         BEGIN
            COSE[J]←COS(THED[J]);
            SINE[J]←SIN(THED[J])
         END;
         FOR I←1 STEP 1 UNTIL N DO
         BEGIN
            FOR J←0 STEP 1 UNTIL ND DO
            BEGIN
               IF XC[I]≥XD[J] THEN TEMP←0 ELSE TEMP←1;
               MI[J]←-HORIZS[I]×YD[J]+VERTS[I]×XD[J]-TEMP×(XD[J]-XC[I]);
               NI[J]←HORIZS[I]×COSE[J]+(VERTS[I]-TEMP)×SINE[J];
               MM[J]←MI[J]×MI[J];
               NN[J]←NI[J]×NI[J]
            END;
            FOR J←0 STEP 1 UNTIL NP1  DO
            BEGIN
               IF AREA[J]>AREA[J+1] THEN A←AREA[J] ELSE A←AREA[J+1];
               IF INERTIA[J]>INERTIA[J+1] THEN
                  IR←INERTIA[J] ELSE IR←INERTIA[J+1];
               H←1728×DELS[J]/(IR×29000);
               N1←10×J;
```

```
            N2←10×(J+1);
            SIMPSON(N1,N2,H,MM,DEFM);
            H←12×DELS[J]/(A×29000);
            SIMPSON(N1,N2,H,NN,DEFN);
            DEFL[I,I]←DEFL[I,I]+DEFM+DEFN
         END;
         FOR K←I+1 STEP 1 UNTIL N DO
         BEGIN
            FOR J←0 STEP 1 UNTIL ND DO
            BEGIN
               IF XC[K]≥XD[J] THEN TEMP←0 ELSE TEMP←1;
               MJ[J]←-HORIZS[K]×YD[J]+VERTS[K]×XD[J]-TEMP×(XD[J]-XC[K]);
               NJ[J]←HORIZS[K]×COSE[J]+(VERTS[K]-TEMP)×SINE[J]
            END;
            FOR J←0 STEP 1 UNTIL ND DO MM[J]←MI[J]×MJ[J];
            FOR J←0 STEP 1 UNTIL ND DO NN[J]←NI[J]×NJ[J];
            FOR J←0 STEP 1 UNTIL NP1  DO
            BEGIN
               IF AREA[J]≥AREA[J+1] THEN A←AREA[J] ELSE A←AREA[J+1];
               IF INERTIA[J]≥INERTIA[J+1] THEN
                  IR←INERTIA[J] ELSE IR←INERTIA[J+1];
               H←1728×DELS[J]/(IR×29000);
               N1←10×J;
               N2←10×(J+1);
               SIMPSON(N1,N2,H,MM,DEFM);
               H←12×DELS[J]/(A×29000);
               SIMPSON(N1,N2,H,NN,DEFN);
               DEFL[I,K]←DEFL[I,K]+DEFM+DEFN
            END;
            DEFL[K,I]←DEFL[I,K]
         END
      END
   END;
PROCEDURE INFLUENCELINES(NC,NP,DEPTH,SPAN,SPANS,XC,X,Y,THETA,AREA,
   INERTIA,TFLG,HORIZS,VERTS,MOM,THRUST,SHEAR,STRESS,LMAXD,DLD);
%  COMPUTES INFLUENCE LINES FOR MOM, THRUST, SHEAR AND STRESS AT ALL
%  CESIGN POINTS WITH LOAD AT COLUMN POINT
   VALUE NC,NP,DEPTH,SPAN,SPANS;
   INTEGER NC,NP;
   REAL DEPTH,SPAN,SPANS;
   REAL ARRAY XC,LMAXD,DLD,HORIZS,VERTS[0],
      X,Y,THETA,ARFA,INERTIA,TFLG[0],
      MOM,THRUST,SHEAR,STRESS[0,0];
   BEGIN
      INTEGER I,J,N1,K;
      REAL A,D,IR,M,N,PHI,STR,TEMP,TF,S,C;
      D←DEPTH;
      N1←NC-1;
      FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         A←AREA[I];
         IR←INERTIA[I];
         TF←TFLG[I];
         PHI←THETA[I];
         S←SIN(PHI);
         C←COS(PHI);
         MOM[I,0]←MOM[I,NC]←THRUST[I,0]←0;
         THRUST[I,NC]←STRESS[I,0]←STRESS[I,NC]←0;
         FOR J←1 STEP 1 UNTIL N1 DO
         BEGIN
            IF XC[J]>X[I] THEN TEMP←0 ELSE TEMP←1;
            MOM[I,J]←-HORIZS[J]×(Y[I]-LMAXD[I]-DLD[I])
```

```
            +VERTS[J]×X[I]+TEMP×(XC[J]-X[I]);
         THRUST[I,J]←HORIZS[J]×C+VERTS[J]×S-TEMP×S;
         IF XC[J]=X[I] AND X[I]<SPANS/2 THEN
            THRUST[I,J]←THRUST[I,J]+S;
         SHEAR[I,J]←-HORIZS[J]×S+VERTS[J]×C-TEMP×C;
         IF XC[J]=X[I] THEN
         BEGIN
            TEMP←C;
            IF ABS(SHEAR[I,J])<ABS(SHEAR[I,J]+TEMP) THEN
               SHEAR[I,J]←SHEAR[I,J]+TEMP
         END;
         M←MOM[I,J];
         N←THRUST[I,J];
         STRESSP(N,M,A,IR,D,TF,STR);
         STRESS[I,J]←STR;
         MOM[I,J]←MOM[I,J]/SPAN;
      END;
   END
END;
PROCEDURE INTERSTATE(GOO,NC,NP,P,SPAN,XC,MAXSTRESS,MINSTRESS,MAXMOM,
   MINMOM,MAXTHRUST,MINTHRUST,MAXSHEAR,MINSHEAR,FC,MAXDEFL,MINDEFL,
   MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
%  COMPUTES STRESS, FORCE AND DEFLECTION AT EACH DESIGN POINT
   VALUE GOO,NC,NP,P,SPAN;
   INTEGER GOO,NC,NP;
   REAL P,SPAN;
   REAL ARRAY XC,MAXSTRESS,MINSTRESS,MAXMOM,MINMOM,MAXTHRUST,MINTHRUST,
      MAXSHEAR,MINSHEAR,FC,MAXDEFL,MINDEFL,DIST[0],
      MOM,THRUST,SHEAR,STRESS,DEFL[0,0];
   BEGIN
      INTEGER I,J,L,N1;
      REAL DEF,F,IMPACT,M,N,R1,R2,S,STR;
      LABEL L1,L2,L3,L4;
      SWITCH SW←L1,L2,L3;
      N1←NC-1;
      IMPACT←50/(SPAN+125);
      IF IMPACT-0.30>0 THEN IMPACT←0.30;
      F←P×(1+IMPACT);
      FOR I←0 STEP 1 UNTIL N1 DO DIST[I]←XC[I+1]-XC[I];
      GO TO SW[GOO];
   L1:FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         L←1;
         STR←STRESS[I,1];
         FOR J←2 STEP 1 UNTIL N1 DO
            IF STR<STRESS[I,J] THEN
            BEGIN
               STR←STRESS[I,J];
               L←J
            END;
         M←MOM[I,L];
         N←THRUST[I,L];
         IF STRESS[I,L-1]≥STRESS[I,L+1] THEN
            BEGIN
               R1←1+(DIST[L-1]-4)/DIST[L-1];
               R2←4/DIST[L-1];
               STR←STR×R1+STRESS[I,L-1]×R2;
               M←M×R1+MOM[I,L-1]×R2;
               N←N×R1+THRUST[I,L-1]×R2
            END
            ELSE
            BEGIN
```

```
            R1←1+(DIST[L]-4)/DIST[L];
            R2←4/DIST[L];
            STR←STR×R1+STRESS[I,L+1]×R2;
            M←M×R1+MOM[I,L+1]×R2;
            N←N×R1+THRUST[I,L+1]×R2
         END;
      MAXSTRESS[I]←F×STR;
      MAXMOM[I]←F×M×SPAN;
      MAXTHRUST[I]←F×N;
      IF MAXSTRESS[I]<0 THEN
         MAXSTRESS[I]←MAXMOM[I]←MAXTHRUST[I]←0;
      L←1;
      STR←STRESS[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF STR>STRESS[I,J] THEN
         BEGIN
            STR←STRESS[I,J];
            L←J
         END;
      M←MOM[I,L];
      N←THRUST[I,L];
      IF STRESS[I,L-1]≤STRESS[I,L+1] THEN
         BEGIN
            R1←1+(DIST[L-1]-4)/DIST[L-1];
            R2←4/DIST[L-1];
            STR←STR×R1+STRESS[I,L-1]×R2;
            M←M×R1+MOM[I,L-1]×R2;
            N←N×R1+THRUST[I,L-1]×R2
         END
         ELSE
         BEGIN
            R1←1+(DIST[L]-4)/DIST[L];
            R2←4/DIST[L];
            STR←STR×R1+STRESS[I,L+1]×R2;
            M←M×R1+MOM[I,L+1]×R2;
            N←N×R1+THRUST[I,L+1]×R2
         END;
      MINSTRESS[I]←F×STR;
      MINMOM[I]←F×M×SPAN;
      MINTHRUST[I]←F×N;
      IF MINSTRESS[I]>0 THEN
         MINSTRESS[I]←MINMOM[I]←MINTHRUST[I]←0
   END;
   GO TO L4;
L2:FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      L←1;
      S←SHEAR[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF S<SHEAR[I,J] THEN
         BEGIN
            S←SHEAR[I,J];
            L←J
         END;
      IF SHEAR[I,L-1]≥SHEAR[I,L+1] THEN
         BEGIN
            R1←1+(DIST[L-1]-4)/DIST[L-1];
            R2←4/DIST[L-1];
            S←S×R1+SHEAR[I,L-1]×R2
         END
         ELSE
         BEGIN
```

```
            R1←1+(DIST[L]-4)/DIST[L];
            R2←4/DIST[L];
            S←S×R1+SHEAR[I,L+1]×R2
         END;
      MAXSHEAR[I]←F×S;
      IF MAXSHEAR[I]<0 THEN MAXSHEAR[I]←0;
      L←1;
      S←SHEAR[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF S>SHEAR[I,J] THEN
         BEGIN
            S←SHEAR[I,J];
            L←J
         END;
      IF SHEAR[I,L-1]≤SHEAR[I,L+1] THEN
         BEGIN
            R1←1+(DIST[L-1]-4)/DIST[L-1];
            R2←4/DIST[L-1];
            S←S×R1+SHEAR[I,L-1]×R2
         END
         ELSE
         BEGIN
            R1←1+(DIST[L]-4)/DIST[L];
            R2←4/DIST[L];
            S←S×R1+SHEAR[I,L+1]×R2
         END;
      MINSHEAR[I]←F×S;
      IF MINSHEAR[I]>0 THEN MINSHEAR[I]←0
   END;
   FOR I←1 STEP 1 UNTIL N1 DO
   BEGIN
      FC[I]←1+(DIST[I-1]-4)/DIST[I-1];
      F←1+(DIST[I]-4)/DIST[I];
      IF FC[I]≥F THEN
         FC[I]←P×FC[I]×(1+IMPACT)
         ELSE
         FC[I]←P×F×(1+IMPACT)
      END;
   GO TO L4;
L3:FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      L←1;
      DEF←DEFL[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF DEF<DEFL[I,J] THEN
         BEGIN
            DEF←DEFL[I,J];
            L←J
         END;
      IF DEFL[I,L-1]≥DEFL[I,L+1] THEN
         BEGIN
            R1←1+(DIST[L-1]-4)/DIST[L-1];
            R2←4/DIST[L-1];
            DEF←DEF×R1+DEFL[I,L-1]×R2
         END
         ELSE
         BEGIN
            R1←1+(DIST[L]-4)/DIST[L];
            R2←4/DIST[L];
            DEF←DEF×R1+DEFL[I,L+1]×R2
         END;
      MAXDEFL[I]←F×DEF;
```

```
        IF MAXDEFL[I]<0 THEN MAXDEFL[I]←0;
        L←1;
        DEF←DEFL[I,1];
        FOR J←2 STEP 1 UNTIL N1 DO
           IF DEF>DEFL[I,J] THEN
           BEGIN
              DEF←DEFL[I,J];
              L←J
           END;
        IF DEFL[I,L-1]≤DEFL[I,L+1] THEN
           BEGIN
              R1←1+(DIST[L-1]-4)/DIST[L-1];
              R2←4/DIST[L-1];
              DEF←DEF×R1+DEFL[I,L-1]×R2
           END
           ELSE
           BEGIN
              R1←1+(DIST[L]-4)/DIST[L];
              R2←4/DIST[L];
              DEF←DEF×R1+DEFL[I,L+1]×R2
           END;
        MINDEFL[I]←F×DEF;
        IF MINDEFL[I]>0 THEN MINDEFL[I]←0
     END;
L4:END;
PROCEDURE TRUCK(GOO,NC,NP,P1,P2,SPAN,XC,MAXSTRESS,MINSTRESS,MAXMOM,
   MINMOM,MAXTHRUST,MINTHRUST,MAXSHEAR,MINSHEAR,FC,MAXDEFL,MINDEFL,
   MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
%  COMPUTES STRESS, FORCE AND DEFLECTION AT EACH DESIGN POINT
   VALUE GOO,NC,NP,P1,P2,SPAN;
   INTEGER GOO,NC,NP;
   REAL P1,P2,SPAN;
   REAL ARRAY XC,MAXSTRESS,MINSTRESS,MAXMOM,MINMOM,MAXTHRUST,MINTHRUST,
      MAXSHEAR,MINSHEAR,FC,MAXDEFL,MINDEFL,DIST[0],
      MOM,THRUST,SHEAR,STRESS,DEFL[0,0];
   BEGIN
      INTEGER I,J,L,N1;
      REAL DEF,F1,F2,F,IMPACT,M,N,R1,R2,R3,R4,R5,R6,R7,R8,S,STR;
      LABEL L1,L2,L3,L4;
      SWITCH SW←L1,L2,L3;
      IMPACT←50/(SPAN+125);
      IF IMPACT-0.30>0 THEN IMPACT←0.30;
      N1←NC-1;
      F1←P1×(1+IMPACT);
      F2←P2×(1+IMPACT);
      FOR I←0 STEP 1 UNTIL N1 DO DIST[I]←XC[I+1]-XC[I];
      GO TO SW[GOO];
   L1:FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         L←1;
         STR←STRESS[I,1];
         FOR J←2 STEP 1 UNTIL N1 DO
            IF STR<STRESS[I,J] THEN
            BEGIN
               STR←STRESS[I,J];
               L←J
            END;
         R1←(DIST[L-1]-14)/DIST[L-1];
         R2←14/DIST[L-1];
         R3←(DIST[L]-14)/DIST[L];
         R4←14/DIST[L];
         R5←(DIST[L-1]-28)/DIST[L-1];
```

```
   R6←28/DIST[L-1];
   R7←(DIST[L]-28)/DIST[L];
   R8←28/DIST[L];
   IF STRESS[I,L-1]≥STRESS[I,L+1] THEN
      BEGIN
         STR←F2×(STRESS[I,L-1]×R2+STR×R1)
            +F1×(STR×(1+R3/4)+STRESS[I,L+1]×R4/4);
         M←F2×(MOM[I,L-1]×R2+MOM[I,L]×R1)
            +F1×(MOM[I,L]×(1+R3/4)+MOM[I,L+1]×R4/4);
         N←F2×(THRUST[I,L-1]×R2+THRUST[I,L]×R1)
            +F1×(THRUST[I,L]×(1+R3/4)+THRUST[I,L+1]×R4/4)
      END
      ELSE
      BEGIN
         STR←F1×(STRESS[I,L-1]×R2/4+STR×(R1/4+1))
            +F2×(STR×R3+STRESS[I,L+1]×R4);
         M←F1×(MOM[I,L-1]×R2/4+MOM[I,L]×(R1/4+1))
            +F2×(MOM[I,L]×R3+MOM[I,L+1]×R4);
         N←F1×(THRUST[I,L-1]×R2/4+THRUST[I,L]×(R1/4+1))
            +F2×(THRUST[I,L]×R3+THRUST[I,L+1]×R4)
      END;
   MAXSTRESS[I]←STR;
IF DIST[L-1]≥28 THEN BEGIN
   STR←F1×STRESS[I,L-1]×(R2+R6/4)+STRESS[I,L]×(F1×(R1+R5/4)+F2);
   IF STR>MAXSTRESS[I] THEN
   BEGIN
      MAXSTRESS[I]←STR;
      M←F1×MOM[I,L-1]×(R2+R6/4)+MOM[I,L]×(F1×(R1+R5/4)+F2);
      N←F1×THRUST[I,L-1]×(R2+R6/4)+THRUST[I,L]×(F1×(R1+R5/4)+F2)
   END
END;
IF DIST[L]≥28 THEN BEGIN
   STR←STRESS[I,L]×(F2+F1×(R3+R7/4))+F1×STRESS[I,L+1]×(R4+R8/4);
   IF STR>MAXSTRESS[I] THEN
   BEGIN
      MAXSTRESS[I]←STR;
      M←MOM[I,L]×(F2+F1×(R3+R7/4))+F1×MOM[I,L+1]×(R4+R8/4);
      N←THRUST[I,L]×(F2+F1×(R3+R7/4))+F1×THRUST[I,L+1]×(R4+R8/4)
   END
END;
   MAXMOM[I]←M×SPAN;
   MAXTHRUST[I]←N;
   IF MAXSTRESS[I]<0 THEN
      MAXSTRESS[I]←MAXMOM[I]←MAXTHRUST[I]←0;
   L←1;
   STR←STRESS[I,1];
   FOR J←2 STEP 1 UNTIL N1 DO
      IF STR>STRESS[I,J] THEN
      BEGIN
         STR←STRESS[I,J];
         L←J
      END;
   R1←(DIST[L-1]-14)/DIST[L-1];
   R2←14/DIST[L-1];
   R3←(DIST[L]-14)/DIST[L];
   R4←14/DIST[L];
   R5←(DIST[L-1]-28)/DIST[L-1];
   R6←28/DIST[L-1];
   R7←(DIST[L]-28)/DIST[L];
   R8←28/DIST[L];
   IF STRESS[I,L-1]≤STRESS[I,L+1] THEN
      BEGIN
```

```
            STR←F2×(STRESS[I,L-1]×R2+STR×R1)
               +F1×(STR×(1+R3/4)+STRESS[I,L+1]×R4/4);
            M←F2×(MOM[I,L-1]×R2+MOM[I,L]×R1)
               +F1×(MOM[I,L]×(1+R3/4)+MOM[I,L+1]×R4/4);
            N←F2×(THRUST[I,L-1]×R2+THRUST[I,L]×R1)
               +F1×(THRUST[I,L]×(1+R3/4)+THRUST[I,L+1]×R4/4)
         END
         ELSE
         BEGIN
            STR←F1×(STRESS[I,L-1]×R2/4+STR×(R1/4+1))
               +F2×(STR×R3+STRESS[I,L+1]×R4);
            M←F1×(MOM[I,L-1]×R2/4+MOM[I,L]×(R1/4+1))
               +F2×(MOM[I,L]×R3+MOM[I,L+1]×R4);
            N←F1×(THRUST[I,L-1]×R2/4+THRUST[I,L]×(R1/4+1))
               +F2×(THRUST[I,L]×R3+THRUST[I,L+1]×R4)
         END;
      MINSTRESS[I]←STR;
   IF DIST[L-1]≥28 THEN BEGIN
      STR←F1×STRESS[I,L-1]×(R2+R6/4)+STRESS[I,L]×(F1×(R1+R5/4)+F2);
      IF STR<MINSTRESS[I] THEN
      BEGIN
         MINSTRESS[I]←STR;
         M←F1×MOM[I,L-1]×(R2+R6/4)+MOM[I,L]×(F1×(R1+R5/4)+F2);
         N←F1×THRUST[I,L-1]×(R2+R6/4)+THRUST[I,L]×(F1×(R1+R5/4)+F2)
      END
   END;
   IF DIST[L]≥28 THEN BEGIN
      STR←STRESS[I,L]×(F2+F1×(R3+R7/4))+F1×STRESS[I,L+1]×(R4+R8/4);
      IF STR<MINSTRESS[I] THEN
      BEGIN
         MINSTRESS[I]←STR;
         M←MOM[I,L]×(F2+F1×(R3+R7/4))+F1×MOM[I,L+1]×(R4+R8/4);
         N←THRUST[I,L]×(F2+F1×(R3+R7/4))+F1×THRUST[I,L+1]×(R4+R8/4)
      END
   END;
      MINMOM[I]←M×SPAN;
      MINTHRUST[I]←N;
      IF MINSTRESS[I]>0 THEN
         MINSTRESS[I]←MINMOM[I]←MINTHRUST[I]←0
   END;
   GO TO L4;
L2:FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      L←1;
      S←SHEAR[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF S<SHEAR[I,J] THEN
         BEGIN
            S←SHEAR[I,J];
            L←J
         END;
      R1←(DIST[L-1]-14)/DIST[L-1];
      R2←14/DIST[L-1];
      R3←(DIST[L]-14)/DIST[L];
      R4←14/DIST[L];
      R5←(DIST[L-1]-28)/DIST[L-1];
      R6←28/DIST[L-1];
      R7←(DIST[L]-28)/DIST[L];
      R8←28/DIST[L];
      IF SHEAR[I,L-1]≥SHEAR[I,L+1] THEN
        S←F2×(SHEAR[I,L-1]×R2+S×R1)+F1×(S×(1+R3/4)+SHEAR[I,L+1]×R4/4)
        ELSE
```

```
      S←F1×(SHEAR[I,L-1]×R2/4+S×(R1/4+1))+F2×(S×R3+SHEAR[I,L+1]×R4);
     MAXSHEAR[I]←S;
   IF DIST[L-1]≥28 THEN BEGIN
      S←F1×SHEAR[I,L-1]×(R2+R6/4)+SHEAR[I,L ]×(F1×(R1+R5/4)+F2);
      IF S>MAXSHEAR[I] THEN MAXSHEAR[I]←S
   END;
   IF DIST[L]≥28 THEN BEGIN
      S←SHEAR[I,L]×(F2+F1×(R3+R7/4))+F1×SHEAR[I,L+1]×(R4+R8/4);
      IF S>MAXSHEAR[I] THEN MAXSHEAR[I]←S
   END;
      IF MAXSHEAR[I]<0 THEN MAXSHEAR[I]←0;
      L←1;
      S←SHEAR[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF S>SHEAR[I,J] THEN
         BEGIN
            S←SHEAR[I,J];
            L←J
         END;
      R1←(DIST[L-1]-14)/DIST[L-1];
      R2←14/DIST[L-1];
      R3←(DIST[L]-14)/DIST[L];
      R4←14/DIST[L];
      R5←(DIST[L-1]-28)/DIST[L-1];
      R6←28/DIST[L-1];
      R7←(DIST[L]-28)/DIST[L];
      R8←28/DIST[L];
      IF SHEAR[I,L-1]≤SHEAR[I,L+1] THEN
        S←F2×(SHEAR[I,L-1]×R2+S×R1)+F1×(S×(1+R3/4)+SHEAR[I,L+1]×R4/4)
        ELSE
       S←F1×(SHEAR[I,L-1]×R2/4+S×(R1/4+1))+F2×(S×R3+SHEAR[I,L+1]×R4);
      MINSHEAR[I]←S;
   IF DIST[L-1]≥28 THEN BEGIN
      S←F1×SHEAR[I,L-1]×(R2+R6/4)+SHEAR[I,L ]×(F1×(R1+R5/4)+F2);
      IF S<MINSHEAR[I] THEN MINSHEAR[I]←S
   END;
   IF DIST[L]≥28 THEN BEGIN
      S←SHEAR[I,L]×(F2+F1×(R3+R7/4))+F1×SHEAR[I,L+1]×(R4+R8/4);
      IF S<MINSHEAR[I] THEN MINSHEAR[I]←S
   END;
      IF MINSHEAR[I]>0 THEN MINSHEAR[I]←0
   END;
   FOR I←1 STEP 1 UNTIL N1 DO
   BEGIN
      FC[I]←F1×(1+(DIST[I-1]-14)/(4×DIST[I-1]))
         +F2×(DIST[I]-14)/DIST[I];
      F←F2×(DIST[I-1]-14)/DIST[I-1]+F1×(1+(DIST[I]-14)/(4×DIST[I]));
      IF F>FC[I] THEN FC[I]←F
   END;
   GO TO L4;
L3:FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      L←1;
      DEF←DEFL[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF DEF<DEFL[I,J] THEN
         BEGIN
            DEF←DEFL[I,J];
            L←J
         END;
      R1←(DIST[L-1]-14)/DIST[L-1];
      R2←14/DIST[L-1];
```

```
        R3←(DIST[L]-14)/DIST[L];
        R4←14/DIST[L];
        R5←(DIST[L-1]-28)/DIST[L-1];
        R6←28/DIST[L-1];
        R7←(DIST[L]-28)/DIST[L];
        R8←28/DIST[L];
        IF DEFL[I,L-1]≥DEFL[I,L+1] THEN
           DEF←F2×(DEFL[I,L-1]×R2+DEF×R1)
              +F1×(DEF×(1+R3/4)+DEFL[I,L+1]×R4/4)
           ELSE
           DEF←F1×(DEFL[I,L-1]×R4/4+DEF×(R1/4+1))
              +F2×(DEF×R3+DEFL[I,L+1]×R4);
        MAXDEFL[I]←DEF;
     IF DIST[L-1]≥28 THEN BEGIN
        DEF←F1×DEFL[I,L-1]×(R2+R6/4)+DEFL[I,L]×(F1×(R1+R5/4)+F2);
        IF DEF>MAXDEFL[I] THEN MAXDEFL[I]←DEF
     END;
     IF DIST[L]≥28 THEN BEGIN
        DEF←DEFL[I,L]×(F2+F1×(R3+R7/4))+F1×DEFL[I,L+1]×(R4+R8/4);
        IF DEF>MAXDEFL[I] THEN MAXDEFL[I]←DEF
     END;
        IF MAXDEFL[I]<0 THEN MAXDEFL[I]←0;
        L←1;
        DEF←DEFL[I,1];
        FOR J←2 STEP 1 UNTIL N1 DO
           IF DEF>DEFL[I,J] THEN
           BEGIN
              DEF←DEFL[I,J];
              L←J
           END;
        R1←(DIST[L-1]-14)/DIST[L-1];
        R2←14/DIST[L-1];
        R3←(DIST[L]-14)/DIST[L];
        R4←14/DIST[L];
        R5←(DIST[L-1]-28)/DIST[L-1];
        R6←28/DIST[L-1];
        R7←(DIST[L]-28)/DIST[L];
        R8←28/DIST[L];
        IF DEFL[I,L-1]≤DEFL[I,L+1] THEN
           DEF←F2×(DEFL[I,L-1]×R2+DEF×R1)
              +F1×(DEF×(1+R3/4)+DEFL[I,L+1]×R4/4)
           ELSE
           DEF←F1×(DEFL[I,L-1]×R4/4+DEF×(R1/4+1))
              +F2×(DEF×R3+DEFL[I,L+1]×R4);
        MINDEFL[I]←DEF;
     IF DIST[L-1]≥28 THEN BEGIN
        DEF←F1×DEFL[I,L-1]×(R2+R6/4)+DEFL[I,L]×(F1×(R1+R5/4)+F2);
        IF DEF<MINDEFL[I] THEN MINDEFL[I]←DEF
     END;
     IF DIST[L]≥28 THEN BEGIN
        DEF←DEFL[I,L]×(F2+F1×(R3+R7/4))+F1×DEFL[I,L+1]×(R4+R8/4);
        IF DEF<MINDEFL[I] THEN MINDEFL[I]←DEF
     END;
        IF MINDEFL[I]>0 THEN MINDEFL[I]←0
     END;
L4:END;
PROCEDURE LANE(GOO,NC,NP,W,WS,P,SPAN,XC,MAXSTRESS,MINSTRESS,MAXMOM,
   MINMOM,MAXTHRUST,MINTHRUST,MAXSHEAR,MINSHEAR,FC,MAXDEFL,MINDEFL,
   DIST);
%  COMPUTES STRESS, FORCE AND DEFLECTION AT EACH DESIGN POINT
   VALUE GOO,NC,NP,W,WS,P,SPAN;
   INTEGER GOO,NC,NP;
```

```
REAL W,WS,P,SPAN;
REAL ARRAY XC,MAXSTRESS,MINSTRESS,MAXMOM,MINMOM,MAXTHRUST,MINTHRUST,
   MAXSHEAR,MINSHEAR,FC,MAXDEFL,MINDEFL,DIST[0];
BEGIN
   INTEGER I,J,L,N1;
   REAL DEF,F,IMPACT,M,N,X1,X2,S,STR,WI;
   LABEL L1,L2,L3,L4;
   SWITCH SW←L1,L2,L3;
   IMPACT←50/(SPAN+125);
   IF IMPACT-0,30>0 THEN IMPACT←0,30;
   F←P×(1+IMPACT);
   WI←W×(1+IMPACT);
   N1←NC-1;
   FOR I←0 STEP 1 UNTIL N1 DO DIST[I]←XC[I+1]-XC[I];
   GO TO SW[GOO];
L1:FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      L←1;
      STR←STRESS[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF STR<STRESS[I,J] THEN
         BEGIN
            STR←STRESS[I,J];
            L←J
         END;
      MAXSTRESS[I]←F×STR;
      MAXMOM[I]←F×MOM[I,L]×SPAN;
      MAXTHRUST[I]←F×THRUST[I,L];
      L←1;
      STR←STRESS[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO
         IF STR>STRESS[I,J] THEN
         BEGIN
            STR←STRESS[I,J];
            L←J
         END;
      MINSTRESS[I]←F×STR;
      MINMOM[I]←F×MOM[I,L]×SPAN;
      MINTHRUST[I]←F×THRUST[I,L];
      STR←M←N←0;
      FOR J←0 STEP 1 UNTIL N1 DO
      BEGIN
         IF STRESS[I,J]≥0 AND STRESS[I,J+1]≥0 THEN
         BEGIN
            STR←STR+0,5×(STRESS[I,J]+STRESS[I,J+1])×DIST[J];
            M←M+0,5×(MOM[I,J]+MOM[I,J+1])×DIST[J];
            N←N+0,5×(THRUST[I,J]+THRUST[I,J+1])×DIST[J]
         END;
         IF STRESS[I,J]>0 AND STRESS[I,J+1]<0 THEN
         BEGIN
            X1←DIST[J]×STRESS[I,J]/(STRESS[I,J]-STRESS[I,J+1]);
            STR←STR+0,5×X1×STRESS[I,J];
            M←M+0,5×X1×MOM[I,J]×(2-X1/DIST[J])
               +0,5×X1×X1×MOM[I,J+1]/DIST[J];
            N←N+0,5×X1×THRUST[I,J]×(2-X1/DIST[J])
               +0,5×X1×X1×THRUST[I,J+1]/DIST[J]
         END;
         IF STRESS[I,J]<0 AND STRESS[I,J+1]>0 THEN
         BEGIN
            X2←DIST[J]×STRESS[I,J+1]/(STRESS[I,J+1]-STRESS[I,J]);
            STR←STR+0,5×X2×STRESS[I,J+1];
            M←M+0,5×X2×X2×MOM[I,J]/DIST[J]
```

```
            +0.5×X2×MOM[I,J+1]×(2-X2/DIST[J]);
         N←N+0.5×X2×X2×THRUST[I,J]/DIST[J]
            +0.5×X2×THRUST[I,J+1]×(2-X2/DIST[J])
      END
   END;
   MAXSTRESS[I]←MAXSTRESS[I]+WI×STR;
   MAXMOM[I]←MAXMOM[I]+WI×M×SPAN;
   MAXTHRUST[I]←MAXTHRUST[I]+WI×N;
   MAXSWM[I]←WS×M×SPAN;
   MAXSWN[I]←WS×N;
   IF MAXSTRESS[I]<0 THEN
      MAXSTRESS[I]←MAXMOM[I]←MAXTHRUST[I]←MAXSWM[I]←MAXSWN[I]←0;
   STR←M←N←0;
   FOR J←0 STEP 1 UNTIL N1 DO
   BEGIN
      IF STRESS[I,J]≤0 AND STRESS[I,J+1]≤0 THEN
      BEGIN
         STR←STR+0.5×(STRESS[I,J]+STRESS[I,J+1])×DIST[J];
         M←M+0.5×(MOM[I,J]+MOM[I,J+1])×DIST[J];
         N←N+0.5×(THRUST[I,J]+THRUST[I,J+1])×DIST[J]
      END;
      IF STRESS[I,J]<0 AND STRESS[I,J+1]>0 THEN
      BEGIN
         X1←DIST[J]×STRESS[I,J]/(STRESS[I,J]-STRESS[I,J+1]);
         STR←STR+0.5×X1×STRESS[I,J];
         M←M+0.5×X1×MOM[I,J]×(2-X1/DIST[J])
            +0.5×X1×X1×MOM[I,J+1]/DIST[J];
         N←N+0.5×X1×THRUST[I,J]×(2-X1/DIST[J])
            +0.5×X1×X1×THRUST[I,J+1]/DIST[J]
      END;
      IF STRESS[I,J]>0 AND STRESS[I,J+1]<0 THEN
      BEGIN
         X2←DIST[J]×STRESS[I,J+1]/(STRESS[I,J+1]-STRESS[I,J]);
         STR←STR+0.5×X2×STRESS[I,J+1];
         M←M+0.5×X2×X2×MOM[I,J]/DIST[J]
            +0.5×X2×MOM[I,J+1]×(2-X2/DIST[J]);
         N←N+0.5×X2×X2×THRUST[I,J]/DIST[J]
            +0.5×X2×THRUST[I,J+1]×(2-X2/DIST[J])
      END
   END;
   MINSTRESS[I]←MINSTRESS[I]+WI×STR;
   MINMOM[I]←MINMOM[I]+WI×M×SPAN;
   MINTHRUST[I]←MINTHRUST[I]+WI×N;
   MINSWM[I]←WS×M×SPAN;
   MINSWN[I]←WS×N;
   IF MINSTRESS[I]>0 THEN
      MINSTRESS[I]←MINMOM[I]←MINTHRUST[I]←MINSWM[I]←MINSWN[I]←0
END;
GO TO L4;
L2:F←F×26/18;
FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   S←SHEAR[I,1];
   FOR J←2 STEP 1 UNTIL N1 DO IF S<SHEAR[I,J] THEN S←SHEAR[I,J];
   MAXSHEAR[I]←F×S;
   S←SHEAR[I,1];
   FOR J←2 STEP 1 UNTIL N1 DO IF S>SHEAR[I,J] THEN S←SHEAR[I,J];
   MINSHEAR[I]←F×S;
   S←0;
   FOR J←0 STEP 1 UNTIL N1 DO
   BEGIN
      IF SHEAR[I,J]≥0 AND SHEAR[I,J+1]≥0 THEN
```

```
            S←S+0.5×(SHEAR[I,J]+SHEAR[I,J+1])×DIST[J];
         IF SHEAR[I,J]>0 AND SHEAR[I,J+1]<0 THEN
            S←S+0.5×SHEAR[I,J]×DIST[J]×SHEAR[I,J]/
               (SHEAR[I,J]-SHEAR[I,J+1]);
         IF SHEAR[I,J]<0 AND SHEAR[I,J+1]>0 THEN
            S←S+0.5×SHEAR[I,J+1]×DIST[J]×SHEAR[I,J+1]/
               (SHEAR[I,J+1]-SHEAR[I,J])
      END;
      MAXSHEAR[I]←MAXSHEAR[I]+WI×S;
      MAXSWV[I]←WS×S;
      IF MAXSHEAR[I]<0 THEN MAXSHEAR[I]←MAXSWV[I]←0;
      S←0;
      FOR J←0 STEP 1 UNTIL N1 DO
      BEGIN
         IF SHEAR[I,J]≤0 AND SHEAR[I,J+1]≤0 THEN
            S←S+0.5×(SHEAR[I,J]+SHEAR[I,J+1])×DIST[J];
         IF SHEAR[I,J]<0 AND SHEAR[I,J+1]>0 THEN
            S←S+0.5×DIST[J]×(SHEAR[I,J]*2)/(SHEAR[I,J]-SHEAR[I,J+1]);
         IF SHEAR[I,J]>0 AND SHEAR[I,J+1]<0 THEN
           S←S+.5×DIST[J]×(SHEAR[I,J+1]*2)/(SHEAR[I,J+1]-SHEAR[I,J])
      END;
      MINSHEAR[I]←MINSHEAR[I]+WI×S;
      MINSWV[I]←WS×S;
      IF MINSHEAR[I]>0 THEN MINSHEAR[I]←MINSWV[I]←0
   END;
   FOR I←1 STEP 1 UNTIL N1 DO
      FC[I]←F+(DIST[I-1]+DIST[I])×WI/2;
   FOR I←1 STEP 1 UNTIL N1 DO
      FCSW[I]←(DIST[I-1]+DIST[I])×WS/2;
   GO TO L4;
L3:FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      DEF←DEFL[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO IF DEF<DEFL[I,J] THEN DEF←DEFL[I,J];
      MAXDEFL[I]←F×DEF;
      DEF←DEFL[I,1];
      FOR J←2 STEP 1 UNTIL N1 DO IF DEF>DEFL[I,J] THEN DEF←DEFL[I,J];
      MINDEFL[I]←F×DEF;
      DEF←0;
      FOR J←0 STEP 1 UNTIL N1 DO
      BEGIN
         IF DEFL[I,J]≥0 AND DEFL[I,J+1]≥0 THEN
            DEF←DEF+0.5×(DEFL[I,J]+DEFL[I,J+1])×DIST[J];
         IF DEFL[I,J]>0 AND DEFL[I,J+1]<0 THEN
            DEF←DEF+0.5×DIST[J]×DEFL[I,J]*2/(DEFL[I,J]-DEFL[I,J+1]);
         IF DEFL[I,J]<0 AND DEFL[I,J+1]>0 THEN
            DEF←DEF+0.5×DIST[J]×DEFL[I,J+1]*2/(DEFL[I,J+1]-DEFL[I,J])
      END;
      MAXDEFL[I]←MAXDEFL[I]+WI×DEF;
      IF MAXDEFL[I]<0 THEN MAXDEFL[I]←0;
      DEF←0;
      FOR J←0 STEP 1 UNTIL N1 DO
      BEGIN
         IF DEFL[I,J]≤0 AND DEFL[I,J+1]≤0 THEN
            DEF←DEF+0.5×(DEFL[I,J]+DEFL[I,J+1])×DIST[J];
         IF DEFL[I,J]<0 AND DEFL[I,J+1]>0 THEN
            DEF←DEF+0.5×DIST[J]×DEFL[I,J]*2/(DEFL[I,J]-DEFL[I,J+1]);
         IF DEFL[I,J]>0 AND DEFL[I,J+1]<0 THEN
            DEF←DEF+0.5×DIST[J]×DEFL[I,J+1]*2/(DEFL[I,J+1]-DEFL[I,J])
      END;
      MINDEFL[I]←MINDEFL[I]+WI×DEF;
      IF MINDEFL[I]>0 THEN MINDEFL[I]←0
```

```
        END;
L4:END;
PROCEDURE RIBDL(NC,NP,SPAN,WEIGHT,FYOPT,M,N,V,D,XC,MOM,THRUST,SHEAR,
     DEFL,XXC);
%   WEIGHT=TOTAL WEIGHT OF RIB
%   M=MOMENT,  N=THRUST
%   THE RIB WGT IS ASSUMED TO BE DISTRIBUTED EVENLY
    VALUE NC,NP,SPAN,WEIGHT,FYOPT;
    INTEGER NC,NP;
    REAL SPAN,WEIGHT,FYOPT;
    REAL ARRAY M,N,V,D,XC,XXC[0],   MOM,THRUST,SHEAR,DEFL[0,0];
    BEGIN
        INTEGER I,J,N1;
        REAL AREA;
        DEFINE X=XXC#;
        X[0]←0;
        X[NC]←SPAN;
        N1←NC-1;
        FOR I←1 STEP 1 UNTIL N1 DO X[I]←XC[I];
        FOR I←0 STEP 1 UNTIL NP DO
        BEGIN
            AREA←0;
            FOR J←0 STEP 1 UNTIL N1 DO
                AREA←AREA+(X[J+1]-X[J])×(MOM[I,J+1]+MOM[I,J])/2/SPAN;
            M[I]←AREA×WEIGHT×SPAN;
            AREA←0;
            FOR J←0 STEP 1 UNTIL N1 DO
                AREA←AREA+(X[J+1]-X[J])×(THRUST[I,J+1]+THRUST[I,J])/2/SPAN;
            N[I]←AREA×WEIGHT
        END;
        IF FYOPT≠0 THEN
        FOR I←0 STEP 1 UNTIL NP DO
        BEGIN
            AREA←0;
            FOR J←0 STEP 1 UNTIL N1 DO
                AREA←AREA+(X[J+1]-X[J])×(SHEAR[I,J+1]+SHEAR[I,J])/2/SPAN;
            V[I]←AREA×WEIGHT;
            AREA←0;
            FOR J←0 STEP 1 UNTIL N1 DO
                AREA←AREA+(X[J+1]-X[J])×(DEFL[I,J+1]+DEFL[I,J])/2/SPAN;
            D[I]←AREA×WEIGHT
        END
    END;
PROCEDURE COLUMNDL(NC,NP,H,SPAN,FYOPT,F,XC,WC,X,Y,MOM,THRUST,SHEAR,
    DEFL,M,N,V,D,YC,FFC);
%   COMPUTES M=MOMENT AND N=THRUST AT NP=DESIGN POINTS DUE TO WEIGHT
%   OF COLUMNS AND DECK
%   NC=NO. OF COLUMNS                       WC=WGT PER FOOT FOR COL
%   FC=FORCE IN COL FROM DECK               XC=HORIZ DIST TO COL
%   M=MOMENT RESULTING                      N=THRUST RESULTING
%   V=SHEAR RESULTING                       D=DEFLECTION RESULTING
%   H=HEIGHT FROM HIGH SUPPORT TO DECK
    VALUE NC,NP,H,SPAN,FYOPT;
    INTEGER NC,NP;
    REAL H,SPAN,FYOPT;
    REAL ARRAY F,XC,WC,X,Y,M,N,V,D,FFC,YC[0],
        MOM,THRUST,SHEAR,DEFL[0,0];
    BEGIN
        INTEGER I,J,K,L;
        DEFINE FC=FFC#;
        LABEL LAB1;
        FOR I←0 STEP 1 UNTIL NP DO M[I]←N[I]←V[I]←D[I]←0;
```

```
      L←NC-1;
      FOR I←1 STEP 1 UNTIL L  DO FC[I]←F[I];
      K←1;
 LAB1:FOR I←1 STEP 1 UNTIL L  DO
      FOR J←K STEP 1 UNTIL NP DO
      BEGIN
         IF XC[I]=X[J] THEN
         BEGIN
            K←J+1;
            YC[I]←Y[J];
            FC[I]←FC[I]+WC[I]×ABS(H-YC[I]);
            GO TO LAB1
         END
      END;
      FOR I←0 STEP 1 UNTIL NP DO
      FOR J←1 STEP 1 UNTIL L DO
      BEGIN
         M[I]←M[I]+FC[J]×MOM[I,J]×SPAN;
         N[I]←N[I]+FC[J]×THRUST[I,J]
      END;
      IF FYOPT≠0 THEN
      FOR I←0 STEP 1 UNTIL NP DO
      FOR J←1 STEP 1 UNTIL L DO
      BEGIN
         V[I]←V[I]+FC[J]×SHEAR[I,J];
         D[I]←D[I]+FC[J]×DEFL[I,J]
      END
   END;
PROCEDURE LF(NP,TYPE,L1,L2,SPAN,SPANS,HEIGHT,DELH,HLF,LONGF,SUM1,X,Y,
   THETA,INERTIA,DELS,XD,YD,LFM,LFN,LFV,SUM);
%  COMPUTES MOMENT AND THRUST DUE TO LONGITUDINAL FORCE
%  HLF=HEIGHT FROM HIGH SUPPORT TO THE LONGITUDINAL FORCE
%  LONGF=LONGITUDINAL FORCE
   VALUE NP,TYPE,L1,L2,SPAN,SPANS,HEIGHT,DELH,HLF,LONGF,SUM1;
   INTEGER NP,TYPE,L1,L2;
   REAL SPAN,SPANS,HEIGHT,DELH,HLF,LONGF,SUM1;
   REAL ARRAY X,Y,THETA,INERTIA,DELS,LFN,LFM,LFV[0],    XD,YD,SUM[0];
   BEGIN
      INTEGER I,J,ND,N1,N2;
      REAL VERT,HLEFT,HRIGHT,X1,X2,H1,H2,XT,YT,RADIUS,H,K,TEMP,
         SUMTEMP,SUM4,SUM5,PHI,ITEMP;
      LABEL LAB1,LAB2,LAB3;
      IF HLF=0 THEN GO TO LAB3;
      IF SPAN-SPANS≤2 THEN
      BEGIN
         HRIGHT←0,5×LONGF;
         GO TO LAB2
      END;
      ND←10×NP;
      SUM4←SUM5←0;
      H1←H2←HLF;
      IF HEIGHT-HLF<0 THEN
      BEGIN
         IF L1=0 OR L2=0 THEN GO TO LAB3;
         X1←X[L1];
         X2←X[L2];
         H1←Y[L1];
         H2←Y[L2];
         GO TO LAB1
      END;
      IF TYPE=1 OR TYPE=2 THEN
      BEGIN
```

```
         K←4×HEIGHT/(SPANS*2);
         XT←SQRT((HEIGHT-HLF)/K);
         X1←SPANS/2-XT;
         X2←SPANS/2+XT
      END;
      IF TYPE=3 OR TYPE=4 THEN
      BEGIN
         RADIUS←SPANS*2/(8×HEIGHT)+HEIGHT/2;
         TEMP←RADIUS-HEIGHT+HLF;
         XT←SQRT(RADIUS*2-TEMP*2);
         X1←SPANS/2-XT;
         X2←SPANS/2+XT
      END;
LAB1:TEMP←LONGF×H1/(2×SPAN);
      FOR I←0 STEP 1 UNTIL NP1  DO
      BEGIN
         IF INERTIA[I]<INERTIA[I+1] THEN
            ITEMP←INERTIA[I+1] ELSE ITEMP←INERTIA[I];
         H←DELS[I]/ITEMP;
         N1←10×I;
         N2←10×(I+1);
         FOR J←N1 STEP 1 UNTIL N2 DO IF XD[J]<X1 THEN
            SUM[J]←(LONGF×YD[J]/2-TEMP×XD[J])×(YD[J]+DELH×XD[J]/SPAN)
            ELSE
            SUM[J]←TEMP×(SPAN-XD[J])×(YD[J]+DELH×XD[J]/SPAN);
         SIMPSON(N1,N2,H,SUM,SUMTEMP);
         SUM4←SUM4+SUMTEMP
      END;
      SUM4←SUM4×144;
      TEMP←LONGF×H2/(2×SPAN);
      FOR I←0 STEP 1 UNTIL NP1  DO
      BEGIN
         IF INERTIA[I]<INERTIA[I+1] THEN
            ITEMP←INERTIA[I+1] ELSE ITEMP←INERTIA[I];
         H←DELS[I]/ITEMP;
         N1←10×I;
         N2←10×(I+1);
         FOR J←N1 STEP 1 UNTIL N2 DO IF XD[J]<X2 THEN
            SUM[J]←(LONGF×YD[J]/2-TEMP×XD[J])×(YD[J]+DELH×XD[J]/SPAN)
            ELSE
            SUM[J]←TEMP×(SPAN-XD[J])×(YD[J]+DELH×XD[J]/SPAN);
         SIMPSON(N1,N2,H,SUM,SUMTEMP);
         SUM5←SUM5+SUMTEMP
      END;
      SUM5←SUM5×144;
      HRIGHT←(SUM4+SUM5)/SUM1;
LAB2:HLEFT←LONGF-HRIGHT;
      VERT←((LONGF×H1+LONGF×H2)/2+HRIGHT×DELH)/SPAN;
      FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         PHI←THETA[I];
         IF X[I]≤X1 THEN
         BEGIN
            LFM[I]←HLEFT×Y[I]-VERT×X[I];
            LFN[I]←-HLEFT×COS(PHI)-VERT×SIN(PHI);
            LFV[I]←HLEFT×SIN(PHI)-VERT×COS(PHI)
         END;
         IF X[I]>X1 AND X[I]<X2 THEN
         BEGIN
            LFM[I]←HLEFT×Y[I]-VERT×X[I]-LONGF×(Y[I]-H1)/2;
            LFN[I]←(LONGF/2-HLEFT)×COS(PHI)-VERT×SIN(PHI);
            LFV[I]←(HLEFT-LONGF/2)×SIN(PHI)-VERT×COS(PHI)
```

```
        END;
        IF X[I]≥X2 THEN
        BEGIN
           LFM[I]←VERT×(SPAN-X[I])-HRIGHT×(DELH+Y[I]);
           LFN[I]←(LONGF-HLEFT)×COS(PHI)-VERT×SIN(PHI);
           LFV[I]←(HLEFT-LONGF)×SIN(PHI)-VERT×COS(PHI)
        END
     END;
LAB3:END;
PROCEDURE TEMPERATURE(NP,SPAN,DELH,TR,TD,FYOPT,SUM1,X,Y,THETA,
   MR,NR,VR,MD,ND,VD);
%  TR FOR TEMPERATURE RISE, TD FOR TEMPERATURE DROP
%  ALFA IS FOR COEFFICIENT OF THERMAL EXPANSION
%  M=MOMENT, N=THRUST
   VALUE NP,SPAN,DELH,TR,TD,FYOPT,SUM1;
   INTEGER NP;
   REAL SPAN,DELH,TR,TD,FYOPT,SUM1;
   REAL ARRAY X,Y,THETA,MR,NR,VR,MD,ND,VD[0];
   BEGIN
     INTEGER I;
     REAL ALFA,E,PHI,R,HORIZS,VERTS;
     ALFA←0.0000065;
     E←29000;
     HORIZS←ALFA×TR×SPAN×E/SUM1;
     VERTS←HORIZS×DELH/SPAN;
     FOR I←0 STEP 1 UNTIL NP DO
     BEGIN
        PHI←THETA[I];
        MR[I]←-HORIZS×Y[I]-VERTS×X[I];
        NR[I]←HORIZS×COS(PHI)-VERTS×SIN(PHI)
     END;
     IF FYOPT≠0 THEN
        FOR I←0 STEP 1 UNTIL NP DO
           VR[I]←-HORIZS×SIN(THETA[I])-VERTS×COS(THETA[I]);
     R←-TD/TR;
     FOR I←0 STEP 1 UNTIL NP DO
     BEGIN
        MD[I]←R×MR[I];
        ND[I]←R×NR[I]
     END;
     IF FYOPT≠0 THEN
        FOR I←0 STEP 1 UNTIL NP DO
           VD[I]←R×VR[I]
   END;
PROCEDURE WIND(NC,NP,SPAN,DEPTH,HD,HC,DECKD,HORIZD,FYOPT,XC,X,Y,DELS,
   MOM,THRUST,SHEAR,WINDM,WINDN,WINDV,WLM,WLN,WLV,DIST,YC,FFC);
%  HD=HEIGHT TO DECK FROM HIGH SUPPORT
%  HC=HEIGHT TO CG OF WIND ON DECK
%  DECKD=DECK DEPTH EXPOSED TO WIND
%  HORIZD=HORIZONTAL DIST BETWEEN ARCH RIBS FOR A TWO RIBED ARCH BRIDGE
   VALUE NC,NP,SPAN,DEPTH,HD,HC,DECKD,HORIZD,FYOPT;
   INTEGER NC,NP;
   REAL SPAN,DEPTH,HD,HC,DECKD,HORIZD,FYOPT;
   REAL ARRAY XC,X,Y,DELS,WINDM,WINDN,WINDV,WLM,WLN,WLV,DIST,FFC,
      YC[0],
      MOM,THRUST,SHEAR[0,0];
   BEGIN
     INTEGER I,J,K,L;
     DEFINE FC=FFC#;
     LABEL L1;
     L←NC-1;
     FOR I←0 STEP 1 UNTIL L DO DIST[I]←XC[I+1]-XC[I];
```

```
    K←1;
 L1:FOR I←1 STEP 1 UNTIL L DO
    FOR J←K STEP 1 UNTIL NP DO IF XC[I]=X[J] THEN
    BEGIN
       K←J+1;
       YC[I]←Y[J];
       GO TO L1
    END;
    FOR I←0 STEP 1 UNTIL NP DO
       WINDM[I]←WINDN[I]←WINDV[I]←WLM[I]←WLN[I]←WLV[I]←0;
    FOR I←1 STEP 1 UNTIL L DO
    BEGIN
       FC[I]←(DIST[I-1]+DIST[I])×(HC-YC[I])×DECKD×.075/(2×HORIZD);
       FC[I]←FC[I]+.075×1.5×1×(HD-YC[I])×ABS(HD-YC[I])/(2×HORIZD);
       FC[I]←0.30×FC[I];
       FC[I]←FC[I]+.075×1.5×1×YC[I]×ABS(HD-YC[I])/(2×HORIZD);
       FC[I]←FC[I]+.075×DEPTH×YC[I]×(DELS[I-1]+DELS[I])×10/(24×HORIZD)
    END;
    FOR I←0 STEP 1 UNTIL NP DO
    FOR J←1 STEP 1 UNTIL L DO
    BEGIN
       WINDM[I]←WINDM[I]+MOM[I,J]×FC[J]×SPAN;
       WINDN[I]←WINDN[I]+THRUST[I,J]×FC[J]
    END;
    IF FYOPT≠0 THEN
       FOR I←0 STEP 1 UNTIL NP DO
       FOR J←1 STEP 1 UNTIL L DO
          WINDV[I]←WINDV[I]+SHEAR[I,J]×FC[J];
    FOR I←1 STEP 1 UNTIL L DO
       FC[I]←.03×(DIST[I-1]+DIST[I])×(HD+6-YC[I])/(2×HORIZD);
    FOR I←0 STEP 1 UNTIL NP DO
    FOR J←1 STEP 1 UNTIL L DO
    BEGIN
       WLM[I]←WLM[I]+MOM[I,J]×FC[J]×SPAN;
       WLN[I]←WLN[I]+THRUST[I,J]×FC[J]
    END;
    IF FYOPT≠0 THEN
       FOR I←0 STEP 1 UNTIL NP DO
       FOR J←1 STEP 1 UNTIL L DO
          WLV[I]←WLV[I]+SHEAR[I,J]×FC[J]
  END;
PROCEDURE EARTHQUAKE(NC,NP,SPAN,HC,HORIZD,C,WD,WR,FYOPT,WC,XC,X,Y,
   DELS,MOM,THRUST,SHEAR,EQM,EQN,EQS,DIST,DELC,YC,FFC);
%  C=EARTHQUAKE COFFF, AASHO SPECS PAGE 25
%  WD=WGT OF DECK IN KIPS/FT
%  WC=WGT OF COLS IN KIPS/FT
%  WR=TOTAL WGT OF RIB IN KIPS
%  EQN=THRUST FORCE DUE TO EARTHQUAKE
   VALUE NC,NP,SPAN,HC,HORIZD,C,WD,WR,FYOPT;
   INTEGER NC,NP;
   REAL SPAN,HC,HORIZD,C,WD,WR,FYOPT;
   REAL ARRAY WC,XC,X,Y,DELS,EQM,EQN,EQS,DIST,DELC,FFC,YC[0],
      MOM,THRUST,SHEAR[0,0];
   BEGIN
      INTEGER I,J,K,L;
      REAL LGTH,WGTR;
      DEFINE FC=FFC#;
      LABEL L1;
      L←NC-1;
      FOR I←0 STEP 1 UNTIL L DO DIST[I]←XC[I+1]-XC[I];
      FOR I←0 STEP 1 UNTIL NP DO
         EQM[I]←EQN[I]←EQS[I]←0;
```

```
      LGTH←0;
      FOR I←0 STEP 1UNTIL NP1  DO LGTH←LGTH+DELS[I];
      LGTH←LGTH×10;
      WGTR←WR/LGTH;
      K←1;
      FOR I←1 STEP 1 UNTIL L DO
      FOR J←K STEP 1 UNTIL NP DO IF XC[I]=X[J] THEN
      BEGIN
         K←J+1;
         YC[I]←Y[J];
         GO TO L1;
   L1:END;
      FOR I←0 STEP 1 UNTIL NC DO DELC[I]←0;
      I←0;
      FOR J←0 STEP 1 UNTIL NP1  DO
      BEGIN
         DELC[I]←DELC[I]+DELS[J];
         IF XC[I+1]=X[J+1] THEN I←I+1
      END;
      FOR I←1 STEP 1 UNTIL L DO
      BEGIN
         FC[I]←(DIST[I-1]+DIST[I])×WD×C×(HC-YC[I])/(2×HORIZD);
         FC[I]←0.30×FC[I];
         FC[I]←WC[I]×C×(HC-YC[I])×ABS(HC-YC[I])/(2×HORIZD)×.65+FC[I];
         FC[I]←FC[I]+(DELC[I-1]+DELC[I])×5×WGTR×YC[I]/HORIZD
      END;
      FOR I←0 STEP 1 UNTIL NP DO
      FOR J←1 STEP 1 UNTIL L DO
      BEGIN
         EQM[I]←EQM[I]+MOM[I,J]×FC[J]×SPAN;
         EQN[I]←EQN[I]+THRUST[I,J]×FC[J]
      END;
      IF FYOPT≠0 THEN
         FOR I←0 STEP 1 UNTIL NP DO
         FOR J←1 STEP 1 UNTIL L DO
            EQS[I]←EQS[I]+SHEAR[I,J]×FC[J]
   END;
PROCEDURE SHEARP(NP,CRITV,LMAXV,LMINV,DLV,LFV,WINDV,WLV,TEMPRV,TEMPDV,
   EQS,MAXV,TMAXV);
%  THIS PROCEDURE COMPUTES THE CRITICAL SHEAR FORCE
VALUE NP;
INTEGER NP;
REAL ARRAY CRITV,LMAXV,LMINV,DLV,LFV,WINDV,WLV,TEMPRV,TEMPDV,EQS,
   MAXV,TMAXV[0];
BEGIN
   INTEGER I;
   FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      DLV[I]←ABS(DLV[I]);
      LFV[I]←ABS(LFV[I]);
      WINDV[I]←ABS(WINDV[I]);
      WLV[I]←ABS(WLV[I]);
      EQS[I]←ABS(EQS[I]);
      TMAXV[I]←TEMPRV[I];
      IF TMAXV[I]<TEMPDV[I] THEN TMAXV[I]←TEMPDV[I];
      IF TMAXV[I]<0 THEN TMAXV[I]←0;
      IF LMAXV[I]<-LMINV[I] THEN LMAXV[I]←-LMINV[I]
   END;
%  GROUP I LOADING
   FOR I←0 STEP 1 UNTIL NP DO
      MAXV[I]←CRITV[I]←DLV[I]+LMAXV[I];
%  GROUP III LOADING
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   FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      MAXV[I]←MAXV[I]+LFV[I]+0.3×WINDV[I]+WLV[I];
      IF CRITV[I]<MAXV[I]/1.25 THEN CRITV[I]←MAXV[I]/1.25;
   END;
%  GROUP VI LOADING
   FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      MAXV[I]←(MAXV[I]+TMAXV[I])/1.40;
      IF CRITV[I]<MAXV[I] THEN CRITV[I]←MAXV[I];
   END;
%  GROUP II LOADING
   FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      MAXV[I]←DLV[I]+WINDV[I];
      IF CRITV[I]<MAXV[I]/1.25 THEN CRITV[I]←MAXV[I]/1.25;
   END;
%  GROUP V LOADING
   FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      MAXV[I]←(MAXV[I]+TMAXV[I])/1.40;
      IF CRITV[I]<MAXV[I] THEN CRITV[I]←MAXV[I];
   END;
%  GROUP IV LOADING
   FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      MAXV[I]←(DLV[I]+LMAXV[I]+TMAXV[I])/1.25;
      IF CRITV[I]<MAXV[I] THEN CRITV[I]←MAXV[I];
   END;
%  GROUP VII LOADING
   FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      MAXV[I]←(DLV[I]+EQS[I])/1.3333;
      IF CRITV[I]<MAXV[I] THEN CRITV[I]←MAXV[I];
   END
END;
PROCEDURE ALSTRESS(FY,FU,CYCLES,ETA,AREA,INERTIA,DEPTH,TFLG,LGTH,
   MAXS,MINS,MG,MS,THRUST,ALLOWABLE);
   VALUE FY,FU,CYCLES,ETA,AREA,INERTIA,DEPTH,TFLG,LGTH,MAXS,MINS,
      MG,MS,THRUST;
   REAL FY,FU,CYCLES,ETA,AREA,INERTIA,DEPTH,TFLG,LGTH,MAXS,MINS,
      MG,MS,THRUST,ALLOWABLE;
   BEGIN
      REAL ALFA,AVES,B,E,EG,ES,FAVE,FRO,K1,PHI,R,RAD,C;
      LABEL L1,L2;
      IF LGTH≠0 THEN
      BEGIN
         C←TFLG+DEPTH/2;
         E←29000;
         EG←MG×12/THRUST;
         RAD←INERTIA/AREA;
         IF FY/ETA<MG×12×C/INERTIA THEN
         BEGIN
            ALLOWABLE←10;
            GO TO L2
         END;
         FAVE←0.45×FY;
      L1:PHI←0.75×12×LGTH×SQRT(ETA×FAVE/(E×RAD));
         PHI←PHI/2;
         AVES←(FY/ETA-MG×12×C/INERTIA)/(1+(0.25+EG×C/RAD)/COS(PHI));
         IF AVES<0 THEN
         BEGIN
```

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            ALLOWABLE←10;
            GO TO L2
         END;
         IF ABS(AVES−FAVE)>0.10 THEN
         BEGIN
            FAVE←AVES;
            GO TO L1
         END;
         ALLOWABLE←AVES+MG ×12×C/INERTIA
      END;
   L2:IF LGTH=0 THEN
      BEGIN
         R←MINS/MAXS;
         IF ABS(R)>1 THEN R←1/R;
         FRO←13.3;
         IF CYCLES= 100000 THEN ALFA←1.06;
         IF CYCLES= 500000 THEN ALFA←0.78;
         IF CYCLES=2000000 THEN ALFA←0.54;
         IF FY≥90 THEN
         BEGIN
            IF CYCLES= 100000 THEN
            BEGIN
               FRO←11.4;
               ALFA←0.65
            END;
            IF CYCLES= 500000 THEN
            BEGIN
               FRO←9.6;
               ALFA←0.23
            END;
            IF CYCLES=2000000 THEN
            BEGIN
               FRO←8.3;
               ALFA←0
            END
         END;
         IF R>0 THEN R←0;
         K1←1+ALFA×(FU/58−1);
         IF K1<1 THEN K1←1;
         ALLOWABLE←0.55×FY/(1−(0.55×FY/(K1×FRO)−1)×R)
      END;
      IF ALLOWABLE>0.55×FY/1.25 THEN ALLOWABLE←0.55×FY/1.25
   END;
PROCEDURE CRITICALSTR(NC,NP,CYCLES,DEPTH,FY,FU,ETA,CRITSTR,CRITM,CRITN,
   ALLOWABLE,AREA,INERTIA,TFLG,DELS,XC,X,LMAXM,LMAXN,LMINM,LMINN,
   DLM,DLN,LFM,LFN,TEMPRM,TEMPRN,TEMPDM,TEMPDN);
%  THIS PROCEDURE COMPUTES CRITICAL STRESS, MOMENT AND THRUST
%  COMPRESSIVE STRESS IS TAKEN AS POSITIVE
   VALUE NC,NP,DEPTH,FY,FU,ETA;
   INTEGER NC,NP;
   REAL DEPTH,FY,FU,ETA;
   REAL ARRAY CRITSTR,CRITM,CRITN,ALLOWABLE,AREA,INERTIA,TFLG,DELS,XC,X,
      LMAXM,LMAXN,LMINM,LMINN,DLM,DLN,LFM,LFN,TEMPRM,TEMPRN,
      TEMPDM,TEMPDN,CYCLES[0];
   BEGIN
      INTEGER I,II,J,K,L;
      REAL A,D,DEL,INER,LGTH,M,N,TF,STR,M1;
      D←DEPTH;
      L←NC−1;
      FOR I←0 STEP 1 UNTIL L DO DELC[I]←0;
      I←0; L←NP−1;
      FOR J←0 STEP 1 UNTIL L DO
```

```
      BEGIN
         DELC[I]←DELC[I]+DELS[J];
         IF XC[I+1]=X[J+1] THEN I←I+1
      END;
      FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         TMAXM[I]←TEMPRM[I];
         TMAXN[I]←TEMPRN[I];
         TMINM[I]←TEMPDM[I];
         TMINN[I]←TEMPDN[I]
      END;
      FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         A←AREA[I];
         INER←INERTIA[I];
         TF←TFLG[I];
         STRESSP(LFN[I],LFM[I],A,INER,D,TF,STR);
         IF STR<0 THEN
         BEGIN
            LFN[I]←-LFN[I];
            LFM[I]←-LFM[I]
         END;
         STRESSP(TMAXN[I],TMAXM[I],A,INER,D,TF,MAXS[I]);
         STRESSP(TMINN[I],TMINM[I],A,INER,D,TF,STR);
         IF STR>MAXS[I] THEN
         BEGIN
            M←TMAXM[I];
            N←TMAXN[I];
            TMAXM[I]←TMINM[I];
            TMAXN[I]←TMINN[I];
            TMINM[I]←M;
            TMINN[I]←N
         END;
         TMINM[I]←TMINN[I]←0;
         STRESSP(WINDN[I],WINDM[I],A,INER,D,TF,STR);
         IF STR<0 THEN
         BEGIN
            WINDM[I]←-WINDM[I];
            WINDN[I]←-WINDN[I]
         END;
         STRESSP(WLN[I],WLM[I],A,INER,D,TF,STR);
         IF STR<0 THEN
         BEGIN
            WLM[I]←-WLM[I];
            WLN[I]←-WLN[I]
         END;
         STRESSP(EQN[I],EQM[I],A,INER,D,TF,STR);
         IF STR<0 THEN
         BEGIN
            EQM[I]←-EQM[I];
            EQN[I]←-EQN[I]
         END
      END;
%  GROUP I LOADING
      FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         A←AREA[I];
         INER←INERTIA[I];
         TF←TFLG[I];
         CRITM[I]←MAXM[I]←DLM[I]+LMAXM[I];
         CRITN[I]←MAXN[I]←DLN[I]+LMAXN[I];
         MINM[I]←DLM[I]+LMINM[I];
```

```
        MINN[I]←DLN[I]+LMINN[I];
        STRESSP(MAXN[I],MAXM[I],A,INER,D,TF,STR);
        CRITSTR[I]←MAXS[I]←STR;
        STRESSP(MINN[I],MINM[I],A,INER,D,TF,STR);
        MINS[I]←STR;
        ALSTRESS(FY,FU,CYCLES[I],ETA,A,INER,D,TF,0,MAXS[I],MINS[I],
           0,0,0,STR);
        ALLOWABLE[I]←STR
    END;
    I←K←0;
    FOR J←K STEP 1 UNTIL NP DO
    BEGIN
        IF XC[I+1]=X[J] OR I=NC-1 THEN
        BEGIN
            IF I=NC-1 THEN J←NP;
            IF CRITSTR[K]>CRITSTR[J] THEN
            BEGIN
                M1←CRITM[J];
                M←CRITM[K];
                N←CRITN[K];
                A←AREA[K];
                INER←INERTIA[K];
                TF←TFLG[K]
            END
            ELSE
            BEGIN
                M1←CRITM[K];
                M←CRITM[J];
                N←CRITN[J];
                A←AREA[J];
                INER←INERTIA[J];
                TF←TFLG[J]
            END;
            LGTH←10×DELC[I];
            ALSTRESS(FY,FU,0,ETA,A,INER,D,TF,LGTH,0,0,M,M1,N,STR);
            FOR II←K STEP 1 UNTIL J DO
                IF STR<ALLOWABLE[II] THEN ALLOWABLE[II]←STR;
            I←I+1;
            K←J
        END
    END;
    FOR I←0 STEP 1 UNTIL NP DO DELSTR[I]←ALLOWABLE[I]-MAXS[I];
%  GROUP III LOADING
    FOR I←0 STEP 1 UNTIL NP DO
    BEGIN
        A←AREA[I];
        INER←INERTIA[I];
        TF←TFLG[I];
        MAXM[I]←MAXM[I]+LFM[I]+0.3×WINDM[I]+WLM[I];
        MAXN[I]←MAXN[I]+LFN[I]+0.3×WINDN[I]+WLN[I];
        MINM[I]←MINM[I]+LFM[I]+0.7×(0.3×WINDM[I]+WLM[I]);
        MINN[I]←MINN[I]+LFN[I]+0.7×(0.3×WINDN[I]+WLN[I]);
        STRESSP(MAXN[I],MAXM[I],A,INER,D,TF,STR);
        MAXS[I]←STR/1.25;
        STRESSP(MINN[I],MINM[I],A,INER,D,TF,STR);
        MINS[I]←STR/1.25;
        ALSTRESS(FY,FU,CYCLES[I],ETA,A,INER,D,TF,0,MAXS[I],MINS[I],
           0,0,0,STR);
        ASTR[I]←STR
    END;
    I←K←0;
    FOR J←K STEP 1 UNTIL NP DO
```

```
BEGIN
   IF XC[I+1]=X[J] OR I=NC-1 THEN
   BEGIN
      IF I=NC-1 THEN J←NP;
      IF MAXS[K]>MAXS[J] THEN
      BEGIN
         M1←MAXM[J]/1.25;
         M←MAXM[K]/1.25;
         N←MAXN[K]/1.25;
         A←AREA[K];
         INER←INERTIA[K];
         TF←TFLG[K]
      END
      ELSE
      BEGIN
         M1←MAXM[K]/1.25;
         M←MAXM[J]/1.25;
         N←MAXN[J]/1.25;
         A←AREA[J];
         INER←INERTIA[J];
         TF←TFLG[J]
      END;
      LGTH←10×DELC[I];
      ALSTRESS(FY,FU,0,ETA,A,INER,D,TF,LGTH,0,0,M,M1,N,STR);
      FOR II←K STEP 1 UNTIL J DO
         IF STR<ASTR[II] THEN ASTR[II]←STR;
      I←I+1;
      K←J
   END
END;
FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   DEL←ASTR[I]-MAXS[I];
   IF DEL<DELSTR[I] THEN
   BEGIN
      DELSTR[I]←DEL;
      CRITSTR[I]←MAXS[I];
      CRITM[I]←MAXM[I]/1.25;
      CRITN[I]←MAXN[I]/1.25;
      ALLOWABLE[I]←ASTR[I]
   END
END;
% GROUP VI LOADING
FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   A←AREA[I];
   INER←INERTIA[I];
   TF←TFLG[I];
   MAXM[I]←MAXM[I]+TMAXM[I];
   MAXN[I]←MAXN[I]+TMAXN[I];
   MINM[I]←MINM[I]+TMAXM[I];
   MINN[I]←MINN[I]+TMAXN[I];
   STRESSP(MAXN[I],MAXM[I],A,INER,D,TF,STR);
   MAXS[I]←STR/1.40;
   STRESSP(MINN[I],MINM[I],A,INER,D,TF,STR);
   MINS[I]←STR/1.40;
   ALSTRESS(FY,FU,CYCLES[I],ETA,A,INER,D,TF,0,MAXS[I],MINS[I],
      0,0,0,STR);
   ASTR[I]←STR
END;
I←K←0;
FOR J←K STEP 1 UNTIL NP DO
```

```
BEGIN
   IF XC[I+1]=X[J] OR I=NC-1 THEN
   BEGIN
      IF I=NC-1 THEN J←NP;
      IF MAXS[K]>MAXS[J] THEN
      BEGIN
         M1←MAXM[J]/1.40;
         M←MAXM[K]/1.40;
         N←MAXN[K]/1.40;
         A←AREA[K];
         INER←INERTIA[K];
         TF←TFLG[K]
      END
      ELSE
      BEGIN
         M1←MAXM[K]/1.40;
         M←MAXM[J]/1.40;
         N←MAXN[J]/1.40;
         A←AREA[J];
         INER←INERTIA[J];
         TF←TFLG[J]
      END;
      LGTH←10×DELC[I];
      ALSTRESS(FY,FU,0,ETA,A,INER,D,TF,LGTH,0,0,M,M1,N,STR);
      FOR II←K STEP 1 UNTIL J DO
         IF STR<ASTR[II] THEN ASTR[II]←STR;
      I←I+1;
      K←J
   END
END;
FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   DEL←ASTR[I]-MAXS[I];
   IF DEL<DELSTR[I] THEN
   BEGIN
      DELSTR[I]←DEL;
      CRITSTR[I]←MAXS[I];
      CRITM[I]←MAXM[I]/1.40;
      CRITN[I]←MAXN[I]/1.40;
      ALLOWABLE[I]←ASTR[I]
   END
END;
%  GROUP II LOADING
FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   A←AREA[I];
   INER←INERTIA[I];
   TF←TFLG[I];
   MAXM[I]←DLM[I]+WINDM[I];
   MAXN[I]←DLN[I]+WINDN[I];
   MINM[I]←DLM[I]+0.7×WINDM[I];
   MINN[I]←DLN[I]+WINDN[I]×0.7;
   STRESSP(MAXN[I],MAXM[I],A,INER,D,TF,STR);
   MAXS[I]←STR/1.25;
   STRESSP(MINN[I],MINM[I],A,INER,D,TF,STR);
   MINS[I]←STR/1.25;
   ALSTRESS(FY,FU,100000,    ETA,A,INER,D,TF,0,MAXS[I],MINS[I],
      0,0,0,STR);
   ASTR[I]←STR
END;
I←K←0;
FOR J←K STEP 1 UNTIL NP DO
```

```
      BEGIN
         IF XC[I+1]=X[J] OR I=NC-1 THEN
         BEGIN
            IF I=NC-1 THEN J←NP;
            IF MAXS[K]>MAXS[J] THEN
            BEGIN
               M1←MAXM[J]/1,25;
               M←MAXM[K]/1,25;
               N←MAXN[K]/1,25;
               A←AREA[K];
               INER←INERTIA[K];
               TF←TFLG[K]
            END
            ELSE
            BEGIN
               M1←MAXM[K]/1,25;
               M←MAXM[J]/1,25;
               N←MAXN[J]/1,25;
               A←AREA[J];
               INER←INERTIA[J];
               TF←TFLG[J]
            END;
            LGTH←10×DELC[I];
            ALSTRESS(FY,FU,0,ETA,A,INER,D,TF,LGTH,0,0,M,M1,N,STR);
            FOR II←K STEP 1 UNTIL J DO
               IF STR<ASTR[II] THEN ASTR[II]←STR;
            I←I+1;
            K←J
         END
      END;
      FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         DEL←ASTR[I]-MAXS[I];
         IF DEL<DELSTR[I] THEN
         BEGIN
            DELSTR[I]←DEL;
            CRITSTR[I]←MAXS[I];
            CRITM[I]←MAXM[I]/1,25;
            CRITN[I]←MAXN[I]/1,25;
            ALLOWABLE[I]←ASTR[I]
         END
      END;
%  GROUP V LOADING
      FOR I←0 STEP 1 UNTIL NP DO
      BEGIN
         A←AREA[I];
         INER←INERTIA[I];
         TF←TFLG[I];
         MAXM[I]←MAXM[I]+TMAXM[I];
         MAXN[I]←MAXN[I]+TMAXN[I];
         MINM[I]←MINM[I]+TMAXM[I];
         MINN[I]←MINN[I]+TMAXN[I];
         STRESSP(MAXN[I],MAXM[I],A,INER,D,TF,STR);
         MAXS[I]←STR/1.40;
         STRESSP(MINN[I],MINM[I],A,INER,D,TF,STR);
         MINS[I]←STR/1.40;
         ALSTRESS(FY,FU,100000,   ETA,A,INER,D,TF,0,MAXS[I],MINS[I],
            0,0,0,STR);
         ASTR[I]←STR
      END;
      I←K←0;
      FOR J←K STEP 1 UNTIL NP DO
```

```
BEGIN
   IF XC[I+1]=X[J] OR I=NC-1 THEN
   BEGIN
      IF I=NC-1 THEN J←NP;
      IF MAXS[K]>MAXS[J] THEN
      BEGIN
         M1←MAXM[J]/1.40;
         M←MAXM[K]/1.40;
         N←MAXN[K]/1.40;
         A←AREA[K];
         INER←INERTIA[K];
         TF←TFLG[K]
      END
      ELSE
      BEGIN
         M1←MAXM[K]/1.40;
         M←MAXM[J]/1.40;
         N←MAXN[J]/1.40;
         A←AREA[J];
         INER←INERTIA[J];
         TF←TFLG[J]
      END;
      LGTH←10×DELC[I];
      ALSTRESS(FY,FU,0,ETA,A,INER,D,TF,LGTH,0,0,M,M1,N,STR);
      FOR II←K STEP 1 UNTIL J DO
         IF STR<ASTR[II] THEN ASTR[II]←STR;
      I←I+1;
      K←J
   END
END;
FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   DEL←ASTR[I]-MAXS[I];
   IF DEL<DELSTR[I] THEN
   BEGIN
      DELSTR[I]←DEL;
      CRITSTR[I]←MAXS[I];
      CRITM[I]←MAXM[I]/1.40;
      CRITN[I]←MAXN[I]/1.40;
      ALLOWABLE[I]←ASTR[I]
   END
END;
%  GROUP IV LOADING
FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   A←AREA[I];
   INER←INERTIA[I];
   TF←TFLG[I];
   MAXM[I]←DLM[I]+LMAXM[I]+TMAXM[I];
   MAXN[I]←DLN[I]+LMAXN[I]+TMAXN[I];
   MINM[I]←DLM[I]+LMINM[I]+TMAXM[I];
   MINN[I]←DLN[I]+LMINN[I]+TMAXN[I];
   STRESSP(MAXN[I],MAXM[I],A,INER,D,TF,STR);
   MAXS[I]←STR/1.25;
   STRESSP(MINN[I],MINM[I],A,INER,D,TF,STR);
   MINS[I]←STR/1.25;
   ALSTRESS(FY,FU,CYCLES[I],ETA,A,INER,D,TF,0,MAXS[I],MINS[I],
      0,0,0,STR);
   ASTR[I]←STR
END;
I←K←0;
FOR J←K STEP 1 UNTIL NP DO
```

```
    BEGIN
       IF XC[I+1]=X[J] OR I=NC-1 THEN
       BEGIN
          IF I=NC-1 THEN J←NP;
          IF MAXS[K]>MAXS[J] THEN
          BEGIN
             M1←MAXM[J]/1,25;
             M←MAXM[K]/1,25;
             N←MAXN[K]/1,25;
             A←AREA[K];
             INER←INERTIA[K];
             TF←TFLG[K]
          END
          ELSE
          BEGIN
             M1←MAXM[K]/1,25;
             M←MAXM[J]/1,25;
             N←MAXN[J]/1,25;
             A←AREA[J];
             INER←INERTIA[J];
             TF←TFLG[J]
          END;
          LGTH←10×DELC[I];
          ALSTRESS(FY,FU,0,ETA,A,INER,D,TF,LGTH,0,0,M,M1,N,STR);
          FOR II←K STEP 1 UNTIL J DO
             IF STR<ASTR[II] THEN ASTR[II]←STR;
          I←I+1;
          K←J
       END
    END;
    FOR I←0 STEP 1 UNTIL NP DO
    BEGIN
       DEL←ASTR[I]-MAXS[I];
       IF DEL<DELSTR[I] THEN
       BEGIN
          DELSTR[I]←DEL;
          CRITSTR[I]←MAXS[I];
          CRITM[I]←MAXM[I]/1,25;
          CRITN[I]←MAXN[I]/1,25;
          ALLOWABLE[I]←ASTR[I]
       END
    END;
%  GROUP VII LOADING
    FOR I←0 STEP 1 UNTIL NP DO
    BEGIN
       A←AREA[I];
       INER←INERTIA[I];
       TF←TFLG[I];
       MAXM[I]←DLM[I]+EQM[I];
       MAXN[I]←DLN[I]+EQN[I];
       STRESSP(MAXN[I],MAXM[I],A,INER,D,TF,STR);
       MAXS[I]←STR/1,3333
    END;
    I←K←0;
    FOR J←K STEP 1 UNTIL NP DO
    BEGIN
       IF XC[I+1]=X[J] OR I=NC-1 THEN
       BEGIN
          IF I=NC-1 THEN J←NP;
          IF MAXS[K]>MAXS[J] THEN
          BEGIN
             M1←MAXM[J]/1,3333;
```

```
            M←MAXM[K]/1.3333;
            N←MAXN[K]/1.3333;
            A←AREA[K];
            INER←INERTIA[K];
            TF←TFLG[K]
         END
         ELSE
         BEGIN
            M1←MAXM[K]/1.3333;
            M←MAXM[J]/1.3333;
            N←MAXN[J]/1.3333;
            A←AREA[J];
            INER←INERTIA[J];
            TF←TFLG[J]
         END;
         LGTH←10×DELC[I];
         ALSTRESS(FY,FU,0,ETA,A,INER,D,TF,LGTH,0,0,M,M1,N,STR);
         FOR II←K STEP 1 UNTIL J DO ASTR[II]←STR;
         I←I+1;
         K←J
      END
   END;
   FOR I←0 STEP 1 UNTIL NP DO
   BEGIN
      DEL←ASTR[I]-MAXS[I];
      IF DEL<DELSTR[I] THEN
      BEGIN
         DELSTR[I]←DEL;
         CRITSTR[I]←MAXS[I];
         CRITM[I]←MAXM[I]/1.3333;
         CRITN[I]←MAXN[I]/1.3333;
         ALLOWABLE[I]←ASTR[I]
      END
   END
END;
PROCEDURE SECTIONADJUST(NP,FY,STRESS,ALLOWABLE,M,N,AREA,INERTIA,DEPTH,
  TWEB,WIDTH,TFLG,TFMIN);
%  THIS PROCEDURE WILL ADJUST THE FLANGE TO SATISFY THE ALLOWABLE
%  N=THRUST, M=MOMENT
  VALUE NP,FY,DEPTH,TWEB,WIDTH,TFMIN;
  INTEGER NP;
  REAL FY,DEPTH,TWEB,WIDTH,TFMIN;
  REAL ARRAY STRESS,ALLOWABLE,M,N,AREA,INERTIA,TFLG[0];
  BEGIN
     INTEGER I;
     REAL D,W,TF,TW,A,INER,STR,EPS,DEL,F1;
     LABEL LAB1,LAB2;
     D←DEPTH;
     W←WIDTH;
     TW←TWEB;
     FOR I←0 STEP 1 UNTIL NP DO
     BEGIN
        TF←TFLG[I];
        A←AREA[I];
        INER←INERTIA[I];
        STRESSP(N[I],M[I],A,INER,D,TF,STR);
   LAB1:STRESS[I]←STR;
        IF M[I]=0 THEN
        BEGIN
           TF←(N[I]/2/ALLOWABLE[I]-D×TW)/W;
           GO TO LAB2
        END;
```

```
            DEL ←ALLOWABLE[I]-STRESS[I];
            F1←-N[I]×2×W/(A×A)+M[I]×12/INER;
            F1←F1-M[I]×12×(D/2+TF)×0.5×W×(D×D+4×D×TF+3×TF×TF)/(INER×INER);
            EPS←DEL/F1;
            TF←TF+EPS;
            AREAP(D,W,TW,TF,A);
            INERTIAP(D,W,TW,TF,INER);
            STRESSP(N[I],M[I],A,INER,D,TF,STR);
            IF TF<TFMIN THEN GO TO LAB2;
            IF ABS(EPS)>0.0001 THEN GO TO LAB1;
            IF ABS(DEL)>0.01  THEN GO TO LAB1;
      LAB2:TFLG[I]←TF;
            AREA[I]←A;
            INERTIA[I]←INER;
            STRESS[I]←STR
         END;
         TF←WIDTH×SQRT(550×FY/1.25)/4000;
         IF TF<TFMIN THEN TF←TFMIN;
         FOR I←0 STEP 1 UNTIL NP DO
         IF TF>TFLG[I] THEN
         BEGIN
            TFLG[I]←TF;
            AREAP(D,W,TW,TF,A);
            INERTIAP(D,W,TW,TF,INER);
            STRESSP(N[I],M[I],A,INER,D,TF,STR);
            AREA[I]←A;
            INERTIA[I]←INER;
            STRESS[I]←STR
         END
      END;
   PROCEDURE COSTP(NP,RIBWGT,STIFFENERS,COST,PRICE1,PRICE2,AREA,DELS);
   %  PRICE1 PER POUND OF STEEL FOR FLANGE AND WEB
   %  PRICE2 PER POUND OF STEEL FOR STIFFENERS AND DIAPHGRAMS
      VALUE NP,PRICE1,PRICE2,STIFFENERS;
      INTEGER NP;
      REAL RIBWGT,STIFFENERS,COST,PRICE1,PRICE2;
      REAL ARRAY AREA,DELS[0];
      BEGIN
         INTEGER I,L;
         REAL VOLUME,ATEMP;
         VOLUME←0;
         L←NP-1;
         FOR I←0 STEP 1 UNTIL L DO
         BEGIN
            IF AREA[I]≤AREA[I+1] THEN ATEMP←AREA[I+1] ELSE ATEMP←AREA[I];
            VOLUME←VOLUME+DELS[I]×ATEMP×10
         END;
         RIBWGT←490×VOLUME/144;
         COST←RIBWGT×PRICE1+STIFFENERS×PRICE2
      END;
      WRITE(PF [NO]);
   %  ALL READ STATEMENTS ARE WITHIN THE NEXT FEW STATEMENTS
   START:READ(CF,/,NC,NP,TARCH,TLOAD,L1,L2,
         SPAN,HEIGHT,DELH,HDECK,HCG,HLF,DECKD,HORIZD,DEPTH,WIDTH,
         PI,PT1,PT2,PL,WL,WS,WDECK,LONGF,TR,TD,RIBWGT,
         FY,FU,ETA,CEQ,PRICE1,PRICE2,
         ALFAK,TOLH,TOLD,TOLDEFL,TOLCOST)[FINISH];
      L←NC-1;
      READ(CF,/,
         FOR I←0 STEP 1 UNTIL NC DO XC[I],
         FOR I←0 STEP 1 UNTIL NP DO X[I],
         FOR I←0 STEP 1 UNTIL NP DO TFLG[I],
```

```
      FOR I←1 STEP 1 UNTIL L DO FC[I],
      FOR I←1 STEP 1 UNTIL L DO WC[I]);
%   INITIALIZATION
    IF ALFAK=0 THEN ALFAK←0.03×SPAN;
    IF TOLH=0 THEN TOLH←ALFAK/3;
    IF TOLD=0 THEN TOLD←ALFAK/3;
    IF TOLDEFL=0 THEN TOLDEFL←0.00125;
    IF TOLCOST=0 THEN TOLCOST←1000;
    NC1←NC-1;    NP1←NP-1;
    ND←10×NP;
    TWMIN←0.3125;   TFMAX←4;
    TFMIN←0.5;
    RATIO←DEPTH/WIDTH;
    STIFFENERS←0.1×RIBWGT;
    STEPK←ALFAK;
    NRIDGE←0;
    DESIGNS←DESIGN←DES←0;
    NO1←1;
    GOMIN←LCC←NEG←1;
    GOSEARCH←ND←DESN←NG←LC←LR←1;
    COPT←HOPT←FYOPT←IDEFL←LDEFL←0;
    GD←1;   GH←GK←0;
    FYY←FY;
    TOLDEFL←TOLDEFL×SPAN;
%   TEMPORARY
TOLDEFL←0.1400;   GOMIN←2;   TLOAD←6;   FYOPT←FY;
LAB1:IF DESIGN≥50 OR DESIGNS≥200 THEN GO TO LAB2;
    DESIGN←DESIGN+1;
    DESIGNS←DESIGNS+1;
    IF DESIGN≥ 46 OR DESIGNS≥196 THEN
    BEGIN
       FYOPT←FY;
       TLOAD←6;
       IF NO1=1 THEN
       BEGIN
          NO1←0;
          LC←DESIGN-1;
          COST←C[LC];
          IF COST>C[DESIGN] AND C[DESIGN]≠0 THEN
          BEGIN LC←DESIGN; COST←C[LC] END;
          IF COST>C[DESIGN-2] THEN BEGIN LC←DESIGN-2; COST←C[LC] END;
          HEIGHT←H[LC];
          DEPTH←D[LC]
       END
    END;
    IF HEIGHT<0.1×SPAN THEN HEIGHT←0.17×SPAN;
    IF DEPTH<0.005×12×SPAN THEN DEPTH←0.020×12×SPAN;
    TWEB←DEPTH×SQRT(550×FY/1.25)/14400;
    IF TWEB<TWMIN THEN TWEB←TWMIN;
    WIDTH←ENTIER(DEPTH/RATIO);
    IF DI[DESIGNS-1]≠0 AND DEPTH≠0 THEN
    BEGIN
    TEMP←DI[DESIGNS-1]/DEPTH;
    TEMP←TEMP×TEMP;
    IF DEPTH≠DI[DESIGNS-1] THEN
       FOR I←0 STEP 1 UNTIL NP DO TFLG[I]←TFLG[I]×TEMP
    END;
    FOR I←0 STEP 1 UNTIL NP DO LMAXS[I]←LMINS[I]←LMAXM[I]←LMINM[I]←
       LMAXN[I]←LMINN[I]←0;
    BEGIN REAL DUMMY;
%   BEGIN ANALYSIS AND DESIGN COMPUTATIONS
    IF TARCH=3 OR TARCH=4 THEN
```

```
BEGIN
   IF DELH=0 THEN SPANL←SPAN ELSE
      SPANL←(SPAN-SQRT((SPAN×SPAN×HEIGHT+HEIGHT×DELH×DELH)/
         (HEIGHT+DELH)))×2×(HEIGHT+DELH)/DELH;
   SPANS←2×SPAN-SPANL;
   RADIUS←HEIGHT/2+SPANS×SPANS/(8×HEIGHT);
   IF RADIUS<HEIGHT+DELH THEN HEIGHT←RADIUS-DELH
END;
IF HEIGHT≠HI[DESIGNS-1] THEN
GEOMETRY(NP,ND,TARCH,SPAN,SPANS,HEIGHT,DELH,X,Y,THETA,DELS,XD,YD,
   THED,LENGTH);
FOR I←0 STEP 1 UNTIL NP DO
   AREAP(DEPTH,WIDTH,TWEB,TFLG[I],AREA[I]);
FOR I←0 STEP 1 UNTIL NP DO
   INERTIAP(DEPTH,WIDTH,TWEB,TFLG[I],INERTIA[I]);
SUPPORTFORCES(NC,NP,SPAN,HEIGHT,DELH,SUM1,AREA,INERTIA,XC,X,XD,Y,YD,
   DELS,HORIZS,VERTS,SUM);
IF GOMIN=2 OR GOMIN=3 OR FYOPT≠0 THEN
BEGIN
   DEFLECTIONS(NC,NP,ND,XC,HORIZS,VERTS,AREA,INERTIA,XD,YD,THED,
      DELS,DEFL,MI,MJ,MM,NI,NJ,NN,COSE,SINE);
   FOR I←0 STEP 1 UNTIL NP DO
   FOR J←0 STEP 1 UNTIL NC DO DEFL[I,J]←DEFL[I,J]/12
END;
INFLUENCELINES(NC,NP,DEPTH,SPAN,SPANS,XC,X,Y,THETA,AREA,INERTIA,
   TFLG,HORIZS,VERTS,MOM,THRUST,SHEAR,STRESS,LMAXD,DLD);
IF TLOAD=1 OR TLOAD=4 OR TLOAD=6 AND PI≠0 THEN
   INTERSTATE(1,NC,NP,PI,SPAN,XC,LMAXS,LMINS,LMAXM,LMINM,LMAXN,LMINN,
      V1,V2,FC1,D1,D2,MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
FOR I←0 STEP 1 UNTIL NP DO CYCLES[I]←100000;
IF TLOAD≠1 OR TLOAD≠3 THEN
BEGIN
   TRUCK(1,NC,NP,PT1,PT2,SPAN,XC,S1,S2,M1,M2,N1,N2,V1,V2,FC1,D1,D2,
      MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
   FOR I←0 STEP 1 UNTIL NP DO IF S1[I]>LMAXS[I] THEN
   BEGIN
      IF PT2=0 THEN CYCLES[I]←2000000 ELSE CYCLES[I]←500000;
      LMAXS[I]←S1[I];
      LMAXM[I]←M1[I];
      LMAXN[I]←N1[I];
      LMINS[I]←S2[I];
      LMINM[I]←M2[I];
      LMINN[I]←N2[I]
END END;
IF TLOAD=3 OR TLOAD=5 OR TLOAD=6 THEN
BEGIN
   LANE(1,NC,NP,WL,WS,PL,SPAN,XC,S1,S2,M1,M2,N1,N2,V1,V2,FC1,
      D1,D2,MAXSWM,MAXSWN,MINSWM,MINSWN,MAXSWV,MINSWV,FCSW,
      MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
   FOR I←0 STEP 1 UNTIL NP DO IF S1[I]>LMAXS[I] THEN
   BEGIN
      CYCLES[I]←100000;
      LMAXS[I]←S1[I];
      LMAXM[I]←M1[I];
      LMAXN[I]←N1[I];
      LMINS[I]←S2[I];
      LMINM[I]←M2[I];
      LMINN[I]←N2[I]
END END;
IF WS≠0 THEN
BEGIN
   FOR I←0 STEP 1 UNTIL NP DO LMAXM[I]←LMAXM[I]+MAXSWM[I];
```

```
   FOR I←0 STEP 1 UNTIL NP DO LMAXN[I]←LMAXN[I]+MAXSWN[I]
END;
WR←RIBWGT+STIFFENERS;
RIBDL(NC,NP,SPAN,WR,FYOPT,DLM,DLN,DLV,DLD,XC,
   MOM,THRUST,SHEAR,DEFL,XXC);
COLUMNDL(NC,NP,HDECK,SPAN,FYOPT,FC,XC,WC,X,Y,MOM,THRUST,SHEAR,DEFL,
   M1,N1,V1,D1,YC,FFC);
FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   DLM[I]←DLM[I]+M1[I];
   DLN[I]←DLN[I]+N1[I]
END;
IF FYOPT≠0 THEN FOR I←0 STEP 1 UNTIL NP DO
BEGIN
   DLV[I]←DLV[I]+V1[I];
   DLD[I]←DLD[I]+D1[I]
END;
IF LONGF≠0 THEN
   LF(NP,TARCH,L1,L2,SPAN,SPANS,HEIGHT,DELH,HLF,LONGF,SUM1,X,Y,THETA,
      INERTIA,DFLS,XD,YD,LFM,LFN,LFV,SUM);
TEMPERATURE(NP,SPAN,DELH,TR,TD,FYOPT,SUM1,X,Y,THETA,TEMPRM,TEMPRN,
   TEMPRV,TEMPDM,TEMPDN,TEMPDV);
WIND(NC,NP,SPAN,DEPTH,HDECK,HCG,DECKD,HORIZD,FYOPT,XC,X,Y,DELS,
   MOM,THRUST,SHEAR,WINDM,WINDN,WINDV,WLM,WLN,WLV,DIST,YC,FFC);
IF CEQ≠0 THEN
   EARTHQUAKE(NC,NP,SPAN,HCG,HORIZD,CEQ,WDECK,WR,FYOPT,WC,XC,X,Y,
      DELS,MOM,THRUST,SHEAR,EQM,EQN,EQS,DIST,DELC,YC,FFC);
CRITICALSTR(NC,NP,CYCLES,DEPTH,FY,FU,ETA,CRITSTR,
   CRITM,CRITN,ALLOWABLE,
   AREA,INERTIA,TFLG,DELS,XC,X,LMAXM,LMAXN,LMINM,LMINN,DLM,DLN,
   LFM,LFN,TEMPRM,TEMPRN,TEMPDM,TEMPDN,
   WINDM,WINDN,WLM,WLN,EQM,EQN,DELC,MAXM,MAXN,MINM,MINN,MAXS,
   MINS,DELSTR,ASTR,TMAXM,TMAXN,TMINM,TMINN);
SECTIONADJUST(NP,FY,CRITSTR,ALLOWABLE,CRITM,CRITN,AREA,INERTIA,
   DEPTH,TWEB,WIDTH,TFLG,TFMIN);
LGTH←0;
L←NP-1;
FOR I←0 STEP 1 UNTIL L  DO LGTH←LGTH+DELS[I];
LGTH←LGTH×10;
STIFFENERS←DEPTH×TWEB×LGTH×0.40×490/144+
   DEPTH×WIDTH×TWEB×LGTH×490/(DEPTH×144×3.4);
COSTP(NP,RIBWGT,STIFFENERS,COST,PRICE1,PRICE2,AREA,DELS);
RIBWGT←RIBWGT/1000;
STIFFENERS←STIFFENERS/1000;
% THIS ENDS THE ANALYSIS AND DESIGN COMPUTATIONS
% COMPUTE LIVE LOAD DEFLECTIONS
IF GOMIN=2 OR GOMIN=3 OR FYOPT≠0 THEN
BEGIN
   IF PI≠0 THEN
   INTERSTATE(3,NC,NP,PI,SPAN,XC,S1,S2,M1,M2,N1,N2,V1,V2,FC1,
      LMAXD,LMIND,MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
   TRUCK(3,NC,NP,PT1,PT2,SPAN,XC,S1,S2,M1,M2,N1,N2,V1,V2,FC1,
      D1,D2,MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
   FOR I←0 STEP 1 UNTIL NP DO
      IF D1[I]>LMAXD[I] THEN LMAXD[I]←D1[I];
   FOR I←0 STEP 1 UNTIL NP DO
      IF D2[I]<LMIND[I] THEN LMIND[I]←D2[I];
   LANE(3,NC,NP,WL,WS,PL,SPAN,XC,S1,S2,M1,M2,N1,N2,V1,V2,FC1,
      D1,D2,MAXSWM,MAXSWN,MINSWM,MINSWN,MAXSWV,MINSWV,FCSW,
      MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
   FOR I←0 STEP 1 UNTIL NP DO
      IF D1[I]>LMAXD[I] THEN LMAXD[I]←D1[I];
```

```
      FOR I←0 STEP 1 UNTIL NP DO
         IF D2[I]<LMIND[I] THEN LMIND[I]←D2[I];
      LDEFL←0;
      FOR I←0 STEP 1 UNTIL NP DO
         IF LDEFL<LMAXD[I] THEN LDEFL←LMAXD[I];
      FOR I←0 STEP 1 UNTIL NP DO
         IF LDEFL<-LMIND[I] THEN LDEFL←-LMIND[I];
      IF LDEFL>TOLDEFL THEN BEGIN GOMIN←2; GOSEARCH←NO←1 END;
   END;
   END OF DUMMY BLOCK;
%  OPTIMIZATION BEGINS HERE
              CI[DESIGNS]←COST;
              DI[DESIGNS]←DEPTH;
              HI[DESIGNS]←HEIGHT;
   DEFLECTION[DESIGN]←DEFLI[DESIGNS]←LDEFL;
   R[DESIGN]←RI[DESIGNS]←RIBWGT;
   S[DESIGN]←SI[DESIGNS]←STIFFENERS;
   IF DI[DESIGNS]≠DI[DESIGNS-1] OR  HI[DESIGNS]≠HI[DESIGNS-1] THEN
   BEGIN
      DESIGN←DESIGN-1;
      DES5←DESIGN;
      GO TO LAB1
   END
   ELSE
      IF ABS(CI[DESIGNS]-CI[DESIGNS-1])>TOLCOST/NG THEN
   BEGIN
      DESIGN←DESIGN-1;
      DES5←DES5+1;
      IF(DES5-DESIGN)≤5 THEN GO TO LAB1
   END;
   C[DESIGN]←CI[DESIGNS];
   D[DESIGN]←DI[DESIGNS];
   H[DESIGN]←HI[DESIGNS];
   IF DESIGN≥46 OR DESIGNS≥196 THEN GO TO LAB1;
   GO TO SEARCH[GOMIN];
CON.:BEGIN
%  METHOD OF CONJUGATE DIRECTIONS BY POWELL-FOX, P 60
   GO TO CSEARCH[GOSEARCH];
CL1:HEIGHT←HEIGHT+STEPK×GH;
   DEPTH←DEPTH+STEPK×GD;
   GOSEARCH←2;
   GO TO LAB1;
CL2:IF COST<C[DESIGN-1] THEN
   BEGIN
      IF DESIGN-LCC>0 AND NO=1 THEN
         STEPK←STEPK×(DESIGN+2-LCC)/(DESIGN+1-LCC);
      NEG←0;
      GO TO CL1
   END;
   NO←0;
   DESIGN←DESIGN-1;
   HEIGHT←H[DESIGN];
   DEPTH←D[DESIGN];
   IF NEG=1 THEN
   BEGIN
      NEG←0;
      GH←-GH;
      GD←-GD;
      IF DESIGN-LCC<2 THEN NO←1;
      GO TO CL1
   END;
   IF STEPK>ALFAK THEN
```

```
BEGIN
   STEPK←STEPK/2;
   NEG←1;
   GO TO CL1
END;
NEG←1;
NG←1;
LCC←DESIGN;
IF GH=0 AND GK=0 THEN
BEGIN
   STEPK←ALFAK;
   IF NG=1 THEN LC←DESIGN;
   GD←0;
   GH←1;
   GOSEARCH←1;
   DESIGN←DESIGN-1;
   GO TO LAB1
END;
IF GD=0 AND GK=0 THEN
BEGIN
   STEPK←ALFAK;
   GK←GD←1;
   GH←0;
   GOSEARCH←1;
   DESIGN←DESIGN-1;
   GO TO LAB1
END;
IF GH≠0 AND GK≠0 THEN
BEGIN
   STEPK←ALFAK;
   GD←1;
   GH←0;
   GOSEARCH←1;
   DESIGN←DESIGN-1;
   GO TO LAB1
END;
GH←H[DESIGN]-H[LC];
GD←D[DESIGN]-D[LC];
GK←SQRT(GH×GH+GD×GD);
LC←LCC←DESIGN;
 IF GK≤0.001 AND ALFAK≤TOLH AND ALFAK≤TOLD THEN GO TO CL3;
IF GK≤0.001 THEN
BEGIN
   GD←GK←0;
   GH←1
END
ELSE
BEGIN
   GH←GH/GK;
   GD←GD/GK
END;
IF GH<0.1 THEN BEGIN GH←GK←0; GD←1 END;
IF GD<0.1 THEN BEGIN GD←GK←0; GH←1 END;
NG←NG+1;
STEPK←ALFAK←ALFAK×(NG-1)/NG;
DESIGN←DESIGN-1;
GOSEARCH←1;
GO TO LAB1;
CL3:GOSEARCH←3;
   FYOPT←FY;
   ILOAD←6;
   GO TO LAB1;
```

```
CL4:DESIGN←DESIGN-1;
   TEMP←C[DESIGN]-COST;
   C[DESIGN]←COST;
   IF ABS(TEMP)>TOLCOST/10 THEN GO TO LAB1;
   GO TO LAB2
END;
RIDGE:BEGIN
%  RIDGE SEARCH IS MADE IN THIS BLOCK
%  THIS SEARCH IS MADE WHEN LIVE LOAD DEFLECTIONS BECOME CRITICAL
   GO TO RSEARCH[GOSEARCH];
RL1:IDEFL←LDEFL;
   GD←SIGN(IDEFL-TOLDEFL);
   GOSEARCH←2;
RL2:DEPTH←DEPTH+STEPK×GD;
   GO TO LAB1;
RL3:IF C[DESIGN]<C[DESIGN-1] OR
   ((IDEFL-LDEFL)>0 AND (LDEFL-TOLDEFL)>0)  THEN
   BEGIN
      IF DESIGN-LR>0 AND NO=1 THEN
         STEPK←STEPK×(DESIGN+1-LR)/(DESIGN-LR);
      IDEFL←LDEFL;
      GO TO RL1
   END;
   NO←0;
   DESIGN←DESIGN-1;
   DEPTH←D[DESIGN];
   IF STEPK>ALFAK/3  THEN
   BEGIN
      STEPK←STEPK/3;
      GO TO RL2
   END;
   STEPK←ALFAK;
   NO←1;
   IF C[LR]<C[DESIGN] AND NRIDGE=1 THEN GO TO RL8;
   IF GH≠0 THEN GO TO RL5 ELSE
   BEGIN
      GOSEARCH←3;
      DESIGN←DESIGN-1;
      GO TO LAB1
   END;
RL4:HEIGHT←HEIGHT+STEPK;
   LR←DESIGN;
   GH←STEPK;
   GOSEARCH←1;
   GO TO LAB1;
RL5:GH←H[DESIGN]-H[LR];
   GD←D[DESIGN]-D[LR];
   GK←SQRT(GH×GH+GD×GD)×SIGN(C[LR]-C[DESIGN]);
   GH←GH/GK;
   GD←GD/GK;
   NO←1;
   IF C[LR]>C[DESIGN] THEN LR←DESIGN ELSE
   BEGIN
      DESIGN←LR;
      HEIGHT←H[DESIGN];
      DEPTH←D[DESIGN]
   END;
   GOSEARCH←4;
RD1:HEIGHT←HEIGHT+STEPK×GH;
   DEPTH←DEPTH+STEPK×GD;
   GO TO LAB1;
RD2:GH←-GH;
```

```
      GD←-GD;
      GO TO RD1;
   RD3:GH←ABS(GD)×SIGN(GH×GD);
      GD←ABS(GH);
      GO TO RD1;
   RL7:IF C[DESIGN]<C[DESIGN-1] OR
         ((IDEFL-LDEFL)>0 AND (LDEFL-TOLDEFL)>0) THEN
      BEGIN
         IF DESIGN-LR>0 AND NO=1 THEN
            STEPK←STEPK×(DESIGN+1-LR)/(DESIGN-LR);
         IDEFL←LDEFL;
         NEG←0;
         GO TO RD1
      END;
      NO←0;
      DESIGN←DESIGN-1;
      DEPTH←D[DESIGN];
      HEIGHT←H[DESIGN];
      IF NEG=1 THEN
      BEGIN
         NEG←0;
         GO TO RD2
      END;
      IF STEPK>ALFAK/3 THEN
      BEGIN
         STEPK←STEPK/3;
         NEG←1;
         GO TO RD1
      END;
      IF LR=DESIGN OR C[LR]≤C[DESIGN] AND NRIDGE=1 THEN GO TO RL8;
      NO←1;
      IF ALFAK≤TOLD AND ALFAK≤TOLH AND
         ABS(C[DESIGN]-C[DESIGN-1] )≤TOLCOST THEN NRIDGE←1;
      NG←NG+1;
      STEPK←ALFAK←(NG-1)×ALFAK/NG;
      STEPK←STEPK/3;
      DESIGN←DESIGN-1;
      IF NRIDGE=1 THEN GOSEARCH←3 ELSE GOSEARCH←2;
      GO TO LAB1;
   RL8:GOSEARCH←5;
      IF LDEFL≤TOLDEFL THEN GO TO RL9;
      IF NRIDGE=1 THEN
      BEGIN
         NRIDGE←0;
         STEPK←ALFAK/3 ;
         GO TO RD3
      END;
      GO TO RD1;
   RL9:DESIGN←DESIGN-1;
      GOSEARCH←6;
      TEMP←C[DESIGN]-COST;
      C[DESIGN]←COST;
      IF LDEFL>TOLDEFL THEN GO TO RL8;
      IF ABS(TEMP)>TOLCOST/10 THEN GO TO LAB1;
      GO TO LAB2
   END;
   CONV:GO TO LAB2;
   LAB2:BEGIN REAL DUMMY;
   %  COMPUTE LIVE LOAD SHEAR FORCE AND COMBINE FOR MAXIMUM SHEAR FORCE
         IF PI≠0 THEN
      INTERSTATE(2,NC,NP,PI,SPAN,XC,S1,S2,M1,M2,N1,N2,LMAXV,LMINV,FCLL,
         D1,D2,MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
```

```
TRUCK(2,NC,NP,PT1,PT2,SPAN,XC,S1,S2,M1,M2,N1,N2,V1,V2,FC1,
   D1,D2,MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
FOR I←0 STEP 1 UNTIL NP DO
   IF V1[I]>LMAXV[I] THEN LMAXV[I]←V1[I];
FOR I←0 STEP 1 UNTIL NP DO
   IF V2[I]<LMINV[I] THEN LMINV[I]←V2[I];
L←NC-1;
FOR I←1 STEP 1 UNTIL L DO
   IF FC1[I]>FCLL[I] THEN FCLL[I]←FC1[I];
LANE(2,NC,NP,WL,WS,PL,SPAN,XC,S1,S2,M1,M2,N1,N2,V1,V2,FC1,
      D1,D2,MAXSWM,MAXSWN,MINSWM,MINSWN,MAXSWV,MINSWV,FCSW,
      MOM,THRUST,SHEAR,STRESS,DEFL,DIST);
FOR I←0 STEP 1 UNTIL NP DO
   IF V1[I]>LMAXV[I] THEN LMAXV[I]←V1[I];
FOR I←0 STEP 1 UNTIL NP DO
   IF V2[I]<LMINV[I] THEN LMINV[I]←V2[I];
FOR I←1 STEP 1 UNTIL L DO
   IF FC1[I]>FCLL[I] THEN FCLL[I]←FC1[I];
IF WS≠0 THEN
BEGIN
   FOR I←0 STEP 1 UNTIL NP DO LMAXV[I]←LMAXV[I]+MAXSWV[I];
   FOR I←0 STEP 1 UNTIL NP DO LMINV[I]←LMINV[I]+MINSWV[I];
   FOR I←1 STEP 1 UNTIL L DO FCLL[I]←FCLL[I]+FCSW[I]
END;
SHEARP(NP,CRITV,LMAXV,LMINV,DLV,LFV,WINDV,WLV,TEMPRV,TEMPDV,
   EQS,MAXV,TMAXV);
END OF DUMMY BLOCK FOR SHEAR;
HOPT←H[DESIGN];
DOPT←D[DESIGN];
BEGIN REAL DUMMY;
%  WRITE FINAL DESIGN AND SEARCH DATA
WRITE(PF,FMT19);
WRITE(PF,FMT21);
FOR I←1 STEP 1 UNTIL DESIGNS DO
   WRITE(PF,FMT36,I,CI[I],DI[I],HI[I],DEFLI[I],RI[I],SI[I]);
WRITE(PF,FMT22);
FOR I←1 STEP 1 UNTIL DESIGN DO
   WRITE(PF,FMT36,I,C[I],D[I],H[I]);
WRITE(PF,FMT1);
WRITE(PF,FMT30,NC,NP,ND,TARCH,L1,L2);
WRITE(PF,FMT33,SPAN,SPANS,LGTH,HEIGHT,DELH,HDECK,HCG,HLF,DECKD,
   HORIZD);
WRITE(PF,FMT2);
WRITE(PF,FMT31,DEPTH,WIDTH,TWEB);
WRITE(PF,FMT3);
WRITE(PF,FMT31,PI,PT1,PT2,PL,WL,WS,WDECK,LONGF,TR,TD,CEQ);
WRITE(PF,FMT4);
WRITE(PF,FMT33,FY,FU,ETA);
WRITE(PF,FMT33,PRICE1,PRICE2);
WRITE(PF,FMT32,ALFAK,TOLH,TOLD,TOLDEFL,TOLCOST);
RIBWGT←RIBWGT×1000;
STIFFENERS←STIFFENERS×1000;
WRITE(PF,FMT5);
WRITE(PF,FMT33,HOPT,DOPT);
WRITE(PF,FMT35,RIBWGT,STIFFENERS,COST);
WRITE(PF,FMT6);
WRITE(PF,FMT33,
   FOR I←0 STEP 1 UNTIL NC DO XC[I]);
WRITE(PF,FMT7);
WRITE(PF,FMT33,
   FOR I←0 STEP 1 UNTIL NP DO X[I]);
WRITE(PF,FMT8);
```

```
WRITE(PF,FMT33,
    FOR I←0 STEP 1 UNTIL NP DO Y[I]);
WRITE(PF,FMT9);
WRITE(PF,FMT31,
    FOR I←0 STEP 1 UNTIL NP DO THETA[I]);
WRITE(PF,FMT10);
WRITE(PF,FMT31,
    FOR I←0 STEP 1 UNTIL NP DO TFLG[I]);
WRITE(PF,FMT11);
WRITE(PF,FMT34,
    FOR I←0 STEP 1 UNTIL NP DO CRITSTR[I]);
WRITE(PF,FMT20);
WRITE(PF,FMT34,
    FOR I←0 STEP 1 UNTIL NP DO ALLOWABLE[I]);
WRITE(PF,FMT12);
WRITE(PF,FMT35,
    FOR I←0 STEP 1 UNTIL NP DO CRITM[I]);
WRITE(PF,FMT13);
WRITE(PF,FMT35,
    FOR I←0 STEP 1 UNTIL NP DO CRITN[I]);
WRITE(PF,FMT14);
WRITE(PF,FMT35,
    FOR I←0 STEP 1 UNTIL NP DO CRITV[I]);
L←NC-1;
WRITE(PF,FMT15);
WRITE(PF,FMT35,
    FOR I←1 STEP 1 UNTIL L DO FC[I]);
WRITE(PF,FMT16);
WRITE(PF,FMT35,
    FOR I←1 STEP 1 UNTIL L DO FCLL[I]);
WRITE(PF,FMT17);
WRITE(PF,FMT31,
    FOR I←0 STEP 1 UNTIL NC DO DLD[I]);
WRITE(PF,FMT18);
WRITE(PF,FMT31,
    FOR I←0 STEP 1 UNTIL NC DO LMAXD[I]);
WRITE(PF,FMT31,
    FOR I←0 STEP 1 UNTIL NC DO LMIND[I]);
END OF DUMMY BLOCK FOR WRITE STATEMENTS;
WRITE(PF [PAGE]);
GO TO START;
FINISH:END.
```

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